BOTTLENECK TRAFFIC CONGESTION UNDER ALTERNATIVE WORK SCHEDULES

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Abstract: This paper employs user equilibrium theory to develop deterministic queueing models to evaluate the queueing phenomena at a single bottleneck under fixed, flexible, and staggered working schedules. Compared with conventional fixed working hours, the results have shown that traffic delay reduced by flexible working hours is in square proportion to the ratio of flexible time duration to peak hour period. Whereas the delay reduced by staggered working hours of uniform-type is linearly proportional to the ratio of staggered time length to peak hour period. The maximum queue length reduced by flexible/staggered working schedules is also linearly proportional to the ratio of flexible or staggered time length to peak hour period. The staggered working hours of step-type can yield even better performance if the number of commuters in each step is managed at a rate equal to the staggered time multiplied by the bottleneck saturation flow rate.

1. INTRODUCTION

Traffic congestion caused by commuters during peak hours is a common nightmare shared by most urban residents. As commuters have fixed starting and off working hours, it inevitably leads to the over-concentration of using the transportation systems at the same period of the day. In order to decentralize the high demand during peak hours, there is an important strategy employed in transportation demand management, which can be accomplished through the following means: (1) adopt flexible working hours; (2) shift the starting/off working schedule (staggered working hours); and (3) levy congestion tolls (Ferguson, 1990). A common goal of these three methods is to induce commuters to change the commuters' departure time from home and the office, thereby resulting in a decentralization of peak hour demand on transportation systems.

Flexible working hour system has been widely implemented by private and governmental organizations worldwide for nearly four decades. This system has yielded positive returns to companies, employees and society at large. It reduces peak-hour traffic and congestion in metropolitan areas, shortens travel times and encourages car pooling (O'Malley, 1975; Jones *et al.*, 1978; Nollen *et al.*, 1978; TRB, 1980). Among its merits include reduced transportation costs, lower air pollution levels, and a deferred demand for investment of transportation infrastructures.

Some investigators have examined the effectiveness of congestion tolls during peak hour (Arnott *et al.*, 1990a, 1993; Laih, 1994; Lan *et al.*, 1987; de Palma *et al.*, 1986; and Braid, 1989). Studying congestion tolls allows one to analyze the extent to which different toll schemes reduce traffic congestion. Vickrey (1969) first used user equilibrium to illustrate the adoption of time-dependent toll, which can completely remove queueing associated with the bottleneck.

Hendrickson *et al.* (1981) examined how schedule delay influences commuter departure time and the formation of congestion. While applying a mathematical formula, Smith (1984) attempted to verify the existence of equilibrium curve of arrivals at a single bottleneck. Daganzo (1985) further confirmed the uniqueness of the equilibrium curve. Newell (1987), Kuwahara (1990), Arnott *et al.* (1990b), Tabuchi (1993) then increased the scope of above assumptions to examine the traffic congestion under different conditions. Those investigations, however, did not address alternative work schedules. The impact of flexible/staggered work schedules on traffic congestion has received limited attention. Although Henderson (1981), D'Este (1985), and Jovanis (1981) developed theoretical models or applied simulation techniques to explore the ability of flexible/staggered work schedules to alleviate traffic congestion, their analytical assumptions differ from those used in the above investigations on congestion tolls and bottleneck equilibrium models. Consequently, their results cannot be easily compared.

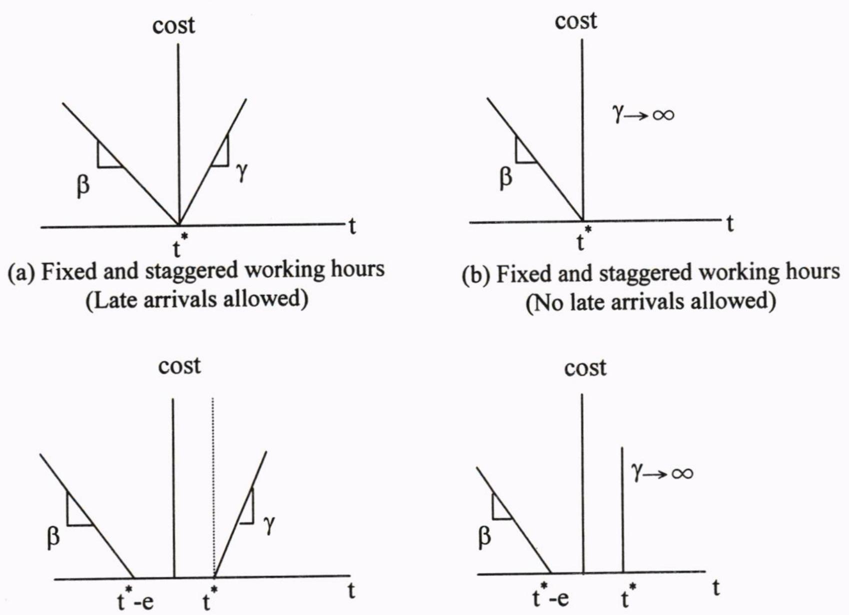
To remedy the gaps of above investigations, this study compares the results obtained from previous efforts to reduce traffic congestion by employing fixed, flexible, and staggered working hours. Different types of work schedules are first defined and, then, a deterministic queueing model is constructed for each work schedule. Measures of effectiveness such as equilibrium travel cost, total queueing delay and maximum queue length are also compared. Sensitivity analysis is also conducted on a given example to discuss policy implications.

2. ASSUMPTIONS

Our analysis will refer to user equilibrium principle applied towards a single bottleneck by following the similar assumptions made by previous studies (e.g. Arnott *et al.*, 1990a; Laih, 1994). (i) All commuters must pass through a bottleneck section before reaching the destination. When the traffic volume exceeds the capacity of bottleneck section, μ, congestion occurs in the bottleneck section, whereas the roads immediately adjacent to the bottleneck that have sufficient capacity are not similarly congested. (ii) The system contains only passenger cars and the occupancy rate remains fixed. (iii) All commuters use the same working hour system. (iv) Each commuter chooses the departure time based on minimum individual travel cost. (v) Commuters follow first-in-first-out principle at the bottleneck section, and the total number of commuters per day is a fixed value. (vi) Travel time between the origin to the bottleneck section and between the bottleneck to the destination is a fixed value and, hence, should not affect the decision on departure time. Therefore, the following discussion omits this factor.

This paper defines three different types of work schedules as follows. Fixed working hours refer to the system where all the employees sign in and leave work at a fixed time, e.g. work starts at 8:30 am to 5:30 pm with an hour lunch break. Staggered working hours refer to a group of workers working in a schedule of fixed hours, and another group of employees working in another schedule of fixed hours. Flexible working hours refer to companies that designate a time range as the core hours where employees must be at work to facilitate communication and business contacts; however, employees can flexibly choose the time to begin and finish work within a given range. Flexible working hour systems are characterized by the notion that if all commuters arrive at work within the length of flexible time, they are viewed as arriving on time. Commuters arriving at the office before their flexible time period begins suffer from early schedule delay; if they arrive after their flexible time period ends, they incur late costs. Commuters encounter the same queueing delay if they arrive within a flexible time period.

We presume that the employees may not all arrive at work on time. They may be early or late; the same applies to getting off work. As the analysis method resembles that of getting to/off work, the discussion below only includes the situation for getting to work. For simplicity, a linear cost function is used to measure the cost for early or late arrivals. The length of early arrivals of commuter i ($h_i(t)$) and the unit cost of time (β) determine the early schedule delay cost. Vice versa, the cost of arriving late at work is calculated from the length of lateness ($p_i(t)$) multiplied by the unit cost of time (γ), usually implying that the employee is subject to penalty. When approaching infinity, the unit cost of time lateness is interpreted as a situation where no lateness is allowed. Let t^* denote starting work time and e the length of flexible time. Figure 1 displays the relationship of schedule delay and cost for early or late arrivals.



(a) Fixed and staggered working hours (Late arrivals allowed)

(b) Fixed and staggered working hours (No late arrivals allowed)

(c) Flexible working hours (Late arrivals allowed)

(d) Flexible working hours (No late arrivals allowed)

Figure 1. Costs for early and late arrivals

The above cost of early and late time plus the fixed cost of travel, ρ, and the cost of queueing delay $\alpha r_i(t)$, equals the function of travel cost for commuter i.

$$U_i(t)=\rho+\alpha r_i(t)+\beta h_i(t)+\gamma p_i(t) \quad (1)$$

As the fixed cost of travel does not influence the analysis results in equation (1), ρ can be set to zero. According to Wardrop's principle, assume that each commuter attempts to reduce the travel cost to a minimum. A situation in which the travel cost is identical for all commuters implies that the commuters' choice of departure time could not reduce the individual travel cost, thereby reaching the state of user equilibrium. By using user equilibrium principle as a basis, the following discussion elaborates on how to perform deterministic queueing analysis to calculate measures of effectiveness (MOE) including equilibrium travel cost, total queueing delay, and maximum queue length in the bottleneck system.

3. THE MODELS

3.1 Fixed Working Hours

In the fixed working hours case, we would follow the previous literature such as Hendrickson *et al.* (1981), Arnott *et al.* (1990a), and Laih (1994) which have utilized user equilibrium mostly under the premise of a fixed working hour system. The travel costs of all commuters, under the condition of user equilibrium, are equal to G_p in late arrivals allowed situation.

$$U_i(t)=\rho+\alpha r_i(t)+\beta h_i(t)+\gamma p_i(t) = G_p \quad \forall\ i \quad (2)$$

Figure 2 depicts the curve of cumulative arrivals A(t) at the bottleneck. At the earliest departure time, t_q commuters experience the longest early schedule delay. While at the latest departure time t_q', commuters incur the largest late penalties; however, at neither point does waiting time in queue exist. At departure time t_j, although commuters suffer maximum queueing delay, there is no early time or late time. According to user equilibrium, the following simultaneous equations are obtained:

$$\beta(x_1+x_2)=G_p$$
$$\alpha x_2=G_p$$
$$\gamma x_3=G_p$$
$$(x_1+x_2+x_3)\mu=N$$

and the solutions are: $x_1=(1-\beta/\alpha)[\gamma/(\beta+\gamma)](N/\mu)$; $x_2=(\beta/\alpha)[\gamma/(\beta+\gamma)](N/\mu)$; $x_3=[\beta/(\beta+\gamma)](N/\mu)$.

Equilibrium travel cost is $G_p=[\beta\gamma/(\beta+\gamma)](N/\mu)$. The slope of the curve of cumulative early arrivals at the bottleneck is $m_1(t)=[\alpha/(\alpha-\beta)]\mu$; whereas the slope of arriving late is $m_2(t)=[\alpha/(\alpha+\gamma)]\mu$. The maximum queue length occurs at $t_j=t_q+\{(1-\beta/\alpha)[\gamma/(\beta+\gamma)](N/\mu)\}$. The area bounded by cumulative arrival and departure curves in Figure 2 is expressed as total queueing delay:

$$W_p = (1/2)\{(\beta\gamma)/[\alpha(\beta+\gamma)]\}(N^2/\mu) \tag{3}$$

Maximum queue length is:

$$Q_p = \{(\beta\gamma)/[\alpha(\beta+\gamma)]\}N. \tag{4}$$

Total queueing delay and maximum queue length are proportional to $(\beta/\alpha)[\gamma/(\beta+\gamma)]$, which are exactly the same as in previous literature (e.g. Hendrickson *et al.*, 1981; Arnott *et al.*, 1990a; Laih, 1994).

With no late arrivals allowed, all commuters must arrive at the office before starting work hour t^*. By using the same method, we can easily obtain $G_p = \beta(N/\mu)$, $W_p = (1/2)(\beta/\alpha)(N^2/\mu)$, and $Q_p = (\beta/\alpha)N$ that are the same as Hendrickson *et al.* (1981).

3.2 Flexible Working Hours

Figure 3 displays the curve A(t) of cumulative arrivals at the bottleneck. Applying the same method of fixed working hours allows us to obtain the simultaneous equations. Notably, equilibrium travel cost is $G_f=[\beta\gamma/(\beta+\gamma)][(N/\mu)-e]$. The number of commuters arriving at the office before flexible time period is $[\gamma/(\beta+\gamma)](N-e\mu)$; arriving within flexible time period is $e\mu$; and lateness is $[\beta/(\beta+\gamma)](N-e\mu)$. The slope of the curve of cumulative early arrivals at the bottleneck is $m_1(t) = [\alpha/(\alpha-\beta)]\mu$, while the similar curve for late arrivals can be expressed as $m_2(t) = [\alpha/(\alpha+\gamma)]\mu$. These arrivals are identical to the model of fixed working hours. The area in Figure 3 bounded by the cumulative arrival and departure curves is expressed as total queueing delay:

$$W_f = (1/2)\{(\beta\gamma)/[\alpha(\beta+\gamma)]\}[(N/\mu)-e](N+e\mu). \tag{5}$$

Maximum queue length is

$$Q_f = \{(\beta\gamma)/[\alpha(\beta+\gamma)]\}(N-e\mu). \tag{6}$$

The measures of effectiveness for situations with no late arrivals allowed can be easily obtained: $G_f = \beta[(N/\mu)-e]$, $W_f = (1/2)(\beta/\alpha)[(N/\mu)-e](N+e\mu)$, and $Q_f = (\beta/\alpha)(N-e\mu)$. Therefore, a longer length of flexible time period implies a larger reduction of total queueing delay as well as maximum queue length in the system. If the length of flexible time period is greater than or equal to peak hour period ($S=N/\mu$), then the queues are eliminated completely.

Comparing the results of fixed and flexible working hours system allows us to reduce total queueing delay in ratio:

$$(W_p - W_f)/W_p = [e/(N/\mu)]^2 = (e/S)^2 \tag{7}$$

On the other hand, equilibrium travel cost or maximum queue length of system reduces in ratio as (e/S).

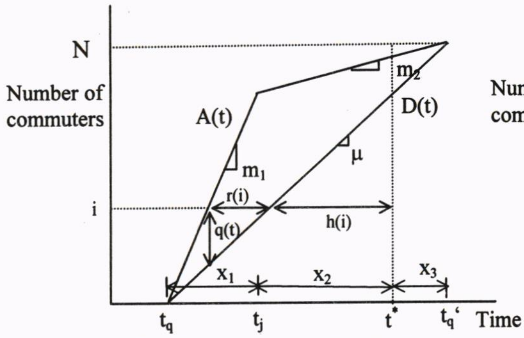


Figure 2. Equilibrium arrival pattern for fixed working hours

[Note] Modification from Hendrickson *et al.* (1981)

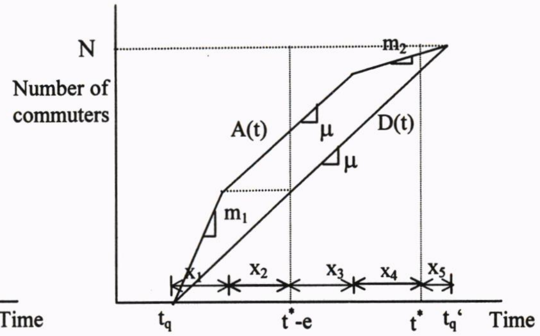


Figure 3. Equilibrium arrival pattern for flexible working hours

3.3 Staggered Working Hours

Two types of staggered working hour systems, step-staggered and uniform-staggered, are discussed herein. When approaching infinity, the number of stages of step-staggered working hours becomes uniform-staggered.

(1) Step type

Two-step staggered working hours are initially discussed and, then, extended to multi-steps. Let one group of workers (vN) choose their starting work hours at t^*-k, and the other group of employees (1-v)N at t^*. The length of staggered time between these two groups is k. Under conditions of user equilibrium and $k \leq [\gamma/(\beta+2\gamma)]S$, Figure 4 displays the curve of cumulative arrivals A(t) at the bottleneck. By utilizing the same method of fixed working hours, the simultaneous equations are obtained. Commuter's travel costs of two groups are the same as $G_s=[\beta\gamma/(\beta+\gamma)][(N/\mu)-k]$. The slope of the curve of cumulative commuters of first group arriving early at the bottleneck is $m_1(t)=m_2(t)=[\alpha/(\alpha-\beta)]\mu$. Meanwhile, the similar curve for the last group of late arrivals is given by $m_3(t) = [\alpha/(\alpha+\gamma)]\mu$. These arrivals are identical to the model of fixed working hours. Compared with fixed working hours, the benefits of reduced queueing delay can be calculated from the shadowed area in Figure 4.

$$\Delta W_s = W_p - W_s = (\beta/\alpha)\{[\gamma/(\beta+\gamma)-v]Nk+[\beta/(\beta+\gamma)]k^2\mu\}. \qquad (8)$$

As generally known, ΔW_s increases when v decreases and ΔW_s reaches maximum when $x_3=0$. Let $x_3=0$, that is $(\beta/\alpha)[(vN/\mu)-k]=0$, we obtain $v=(k\mu/N)$ implying that we can obtain the maximum reduction in queueing delay, if the commuters of first group are $k\mu$ and the others are $N-k\mu$ when k is given. Total queueing delay under such optimal two-step staggered working hours is

$$W_s^* = (1/2)(\beta/\alpha)\{k^2\mu+[\gamma/(\beta+\gamma)](N/\mu-k)^2\mu\}. \qquad (9)$$

Maximum queue length occurs at $t^*-\{\beta\gamma/[\alpha(\beta+\gamma)]\}[(N/\mu)-k$,

$$Q_s^*=\{\beta\gamma/[\alpha(\beta+\gamma)]\}(N-k\mu). \tag{10}$$

If no late arrivals are allowed, when $k\leq S/2$, we can get optimal situation if the commuters of first group are $k\mu$ and last group $N-k\mu$. The measures of effectiveness in the system $G_s=\beta[(N/\mu)-k]$, $W_s^*=(1/2)(\beta/\alpha)[(N^2/\mu)-2(N-k\mu)k]$, $Q_s^*=(\beta/\alpha)(N-k\mu)$ can be obtained easily.

Comparing the results of fixed and optimal two-steps staggered working hours system allows us to reduce the total queueing delay in ratio:

$$(W_p - W_s^*)/ W_p =[1+(\beta/\gamma)](k/S)^2+[1-(k/S)]^2. \tag{11}$$

On the other hand, equilibrium travel cost or maximum queue length of system reduces in ratio as (k/S). If k changes, we set first order condition of equation (11) equal zero and get $k= [\gamma/(\beta+2\gamma)]S$. (If $\gamma\rightarrow\infty$, we obtain $k=S/2$). This implies that maximum benefits can be obtained from two-step staggered working hours when $k=[\gamma/(\beta+2\gamma)]S$. If k is over $[\gamma/(\beta+2\gamma)]S$, some commuters of the first group are late and the total queueing delay is reduced. If k is over S/2, the two groups do not interact with each other and idle time occurs during two peak periods.

Applying the same approach allows us to obtain maximum benefits in three-step staggered working hours if the commuters of first group are $k_1\mu$, second group $k_2\mu$ and last group $N-(k_1+k_2)\mu$. If we extend to multiple-step staggered working hours, the group commuters should be arranged by $k_i\mu$. Consequently, the optimal situation is obtained.

(2) Uniform type

For uniform type staggered system, commuters accumulating by starting work time are arranged as a line with the slope $\omega(\omega>\mu)$. The first commuter starting work time is t_0 and the last starting work t^*. The time length between them is $d= t^*-t_0=N/\omega$. Figure 5 depicts the curve A(t) of cumulative arrivals at the bottleneck. At the earliest departure time, t_q, commuters have the longest early schedule delay. Meanwhile, at the latest departure time, t_q', commuters incur the largest late penalties; however, at both points, no queueing delays exist. Applying the same method allows us to obtain the simultaneous equations. Equilibrium travel cost is $G_u = [\beta\gamma/(\beta+\gamma)](N/\mu)(1-\mu/\omega)$. The slope of the curve of cumulative early arrivals at the bottleneck is $m_1(t)=\mu/[1-(\beta/\alpha)(1-\mu/\omega)]$; similarly for late arrivals $m_2(t)=\mu/[1+(\gamma/\alpha)(1-\mu/\omega)]$, both slopes are smaller than those of fixed working hours. The number of early arrivals are $[\gamma/(\beta+\gamma)]N$ and late arrivals are $[\beta/(\beta+\gamma)]N$. The area in Figure 5 bounded by the cumulative arrival and departure curves is expressed as total queueing delay:

$$W_u=(1/2)\{(\beta\gamma)/[\alpha(\beta+\gamma)]\}(N^2/\mu)(1-\mu/\omega). \tag{12}$$

Maximum queue length :

$$Q_u=\{(\beta\gamma)/[\alpha(\beta+\gamma)]\}N(1-\mu/\omega). \tag{13}$$

According to above equations, total queueing delay and maximum queue length are lower when ω gets smaller, or equivalently, when the length of staggered hours ($d=N/\omega$) is larger. With no late arrivals allowed, measures of effectiveness $G_u = \beta(N/\mu)(1-\mu/\omega)$, $W_u = (1/2)(\beta/\alpha)(N^2/\mu)(1-\mu/\omega)$, and $Q_u = (\beta/\alpha)N(1-\mu/\omega)$ are obtained. If the length of staggered hours is longer than the length of peak period, queueing delay vanishes completely as well.

By comparing the results of fixed and uniform-type staggered working hours, total queueing delay reduces in ratio:

$$(W_p - W_u)/W_p = \mu/\omega = (N/\omega)/(N/\mu) = d/S. \tag{14}$$

Equilibrium travel cost or maximum queue length of system reduces in ratio also as (d/S). Table 1 summarizes the final results of alternative work schedules. Table 2 lists the benefits of flexible and staggered working hours, compared with fixed working hours.

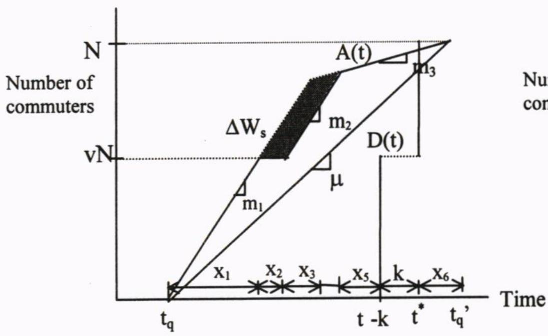


Figure 4. Equilibrium arrival pattern of staggered working hours of two-step type

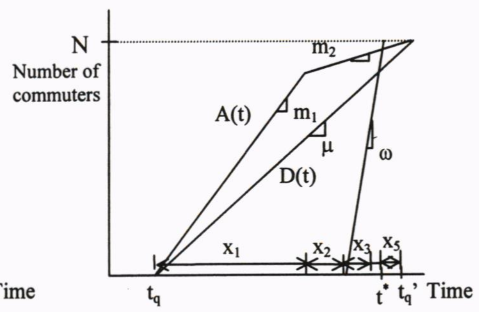


Figure 5. Equilibrium arrival pattern of staggered working hours of uniform type

Table 1. Measures of effectiveness under alternative work schedules

Work schedules	Travel cost	Total queueing delay	Max. queue length
• Fixed working hours	$[\beta\gamma/(\beta+\gamma)](N/\mu)$	$(1/2)\{(\beta\gamma)/[\alpha(\beta+\gamma)]\}(N^2/\mu)$	$\{(\beta\gamma)/[\alpha(\beta+\gamma)]\}N$
• Flexible working hours	$[\beta\gamma/(\beta+\gamma)][(N/\mu)-e]$	$(1/2)\{(\beta\gamma)/[\alpha(\beta+\gamma)]\}[(N/\mu)-e](N+e\mu)$	$\{(\beta\gamma)/[\alpha(\beta+\gamma)]\}(N-e\mu)$
• Uniform staggered working hours	$[\beta\gamma/(\beta+\gamma)](N/\mu)(1-\mu/\omega)$	$(1/2)\{(\beta\gamma)/[\alpha(\beta+\gamma)]\}(N^2/\mu)\times(1-\mu/\omega)$	$\{(\beta\gamma)/[\alpha(\beta+\gamma)]\}N(1-\mu/\omega)$
• Optimal two-step staggered working hours	$[\beta\gamma/(\beta+\gamma)][(N/\mu)-k]$	$(1/2)(\beta/\alpha)\{k^2\mu+[\gamma/(\beta+\gamma)]\times(N/\mu-k)^2\mu\}$	$\{\beta\gamma/[\alpha(\beta+\gamma)]\}(N-k\mu)$

Table 2. Benefits of flexible and staggered compared with fixed working hours

Work schedules	Travel cost reduction	Total queueing delay reduction	Max. queue length reduction
• Flexible working hours	e/S	$(e/S)^2$	e/S
• Uniform staggered working hours	d/S	d/S	d/S
• Optimal two-step staggered working hours	k/S	$\left[1+\frac{\beta}{\gamma}\right]\frac{k^2}{S^2}+\left(1-\frac{k}{S}\right)^2$	k/S

4. DISCUSSIONS

4.1 Multi-Step versus Uniform Staggered Working Hours

In two identical steps staggered working system, the number of commuters in first and last groups equals N/2. The time length between two steps is k. Figure 6 depicts the curve of cumulative arrivals at the bottleneck. According to the results of 3.3, equilibrium travel cost is $\beta[(N/\mu)-k]$ and total queueing delay is $(1/2)(\beta/\alpha)[(N^2/\mu)-kN]$. Since $k=N/\omega$, equilibrium travel cost and total queueing delay can be expressed as $\beta[(N/\mu)-k]$ and $(1/2)(\beta/\alpha)(N^2/\mu)(1-\mu/\omega)$, which are the same as those of staggered working hours of uniform type.

In three identical steps, the number of commuters in first, second and last group equal N/3; each time segment between groups equals k/2. By employing the same method, equilibrium travel cost is $\beta[(N/\mu)-k]$ and total queueing delay is $(1/2)(\beta/\alpha)(N^2/\mu)(1-\mu/\omega)$. In multiple identical steps, the number of commuters in each group is N/η and each time segment between groups equals $k/(\eta-1)$. Under equilibrium conditions, travel cost and total queueing delay are all independent of η, which are the same as those of staggered working hours of uniform type. Namely, if $\eta\rightarrow\infty$, the type of staggered working hours of multiple identical steps becomes uniform type.

We are interested in multi-step staggered working hours system when the number of steps approaches infinity under optimal situation in which total queueing delay is a minimum. From the results of 3.3 for optimal two steps, equilibrium travel cost is $\beta[(N/\mu)-k]$ and total queueing delay is $(1/2)(\beta/\alpha)[(N^2/\mu)-(2Nk-2k^2\mu)]$. In optimal three steps, each time segment between groups equals k/2. The number of commuters in first and second groups are equal, with $(1/2)(k\mu)$, and last group is $N-k\mu$. Although equilibrium travel cost is also $\beta[(N/\mu)-k]$, total queueing delay is reduced as $(1/2)(\beta/\alpha)\{(N^2/\mu)-[2Nk-(3/2)k^2\mu]\}$. In optimal multiple steps case, each time segment between groups equals $k/(\eta-1)$ and the number of first n-1 groups are equal, with $[1/(\eta-1)](k\mu)$, and last group being $N-k\mu$. Equilibrium travel cost and total queueing delay are $\beta[(N/\mu)-k]$ and $(1/2)(\beta/\alpha)\{(N^2/\mu)-[2Nk-(\eta/(\eta-1))k^2\mu]\}$. If $\eta\rightarrow\infty$, total queueing delay is $(1/2)(\beta/\alpha)[(N^2/\mu)-2Nk+k^2\mu]$ and the curve of cumulative arrivals at the bottleneck is shown in Figure7. In this case, the group of commuters $k\mu$ must be arranged with a slope μ, complying with the saturation rate of the bottleneck section. Meanwhile, starting work of the other group of commuters $N-k\mu$ is arranged at time t^*. Thus, it can be viewed as $N-k\mu$ commuters passing through the bottleneck.

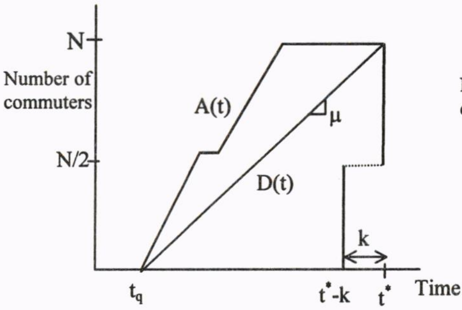


Figure 6. Equilibrium arrival pattern of staggered working hours of two identical steps

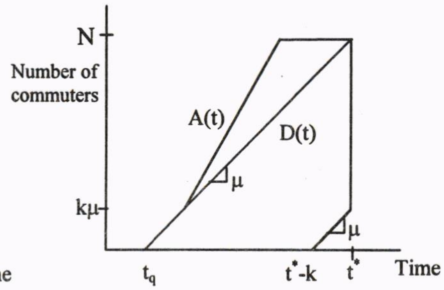


Figure 7. Equilibrium arrival pattern of staggered working hours of optimal multiple steps ($n \to \infty$)

4.2 Relationship Among Alternative Work Schedules

Figure 8 presents an overall framework of alternative work schedules. It depicts that staggered working hours of step type and flexible working hours are two fundamental types, and the other systems should be viewed as special cases. The case of no late arrivals allowed is discussed, for simplicity, as follows:

(1) Staggered working hours of step type

Multiple identical steps is one of staggered working hours of step type where the number of commuters in each step is equal to each other. When the number of steps, η, approaches infinity, total queueing delay is $(1/2)(\beta/\alpha)[(N^2/\mu)-kN]$ which is independent of η. When $\eta=2$, it is two identical-step type. When $\eta \to \infty$, it becomes uniform type in which total queueing delay is $(1/2)(\beta/\alpha)(N^2/\mu)(1-\mu/\omega)$ because $k=N/\omega$. When $\omega \to \infty$, cumulative rate of starting work is near vertical which becomes fixed working hours with total queueing delay being $(1/2)(\beta/\alpha)(N^2/\mu)$.

If the number of steps remains constant in staggered working hours of step type, total queueing delay of optimal multiple steps should be minimum. Total queueing delay is $(1/2)(\beta/\alpha)\{(N^2/\mu)-[2Nk-(\eta/(\eta-1))k^2\mu]\}$. If $\eta=2$, it is optimal two steps type in which total queueing delay is $(1/2)(\beta/\alpha)[(N^2/\mu)-2k(N-k\mu)]$. Furthermore if $k=0$, a fixed working hours case is obtained with total queueing delay $(1/2)(\beta/\alpha)(N^2/\mu)$.

(2) Flexible working hours

In flexible working hours case, all commuters arrive on time if each arrives at work within the length of flexible time, e. Total queueing delay is $(1/2)(\beta/\alpha)[(N/\mu)-e](N+e\mu)$. When $e=0$, it becomes a fixed working hours case in which total queueing delay is $(1/2)(\beta/\alpha)(N^2/\mu)$.

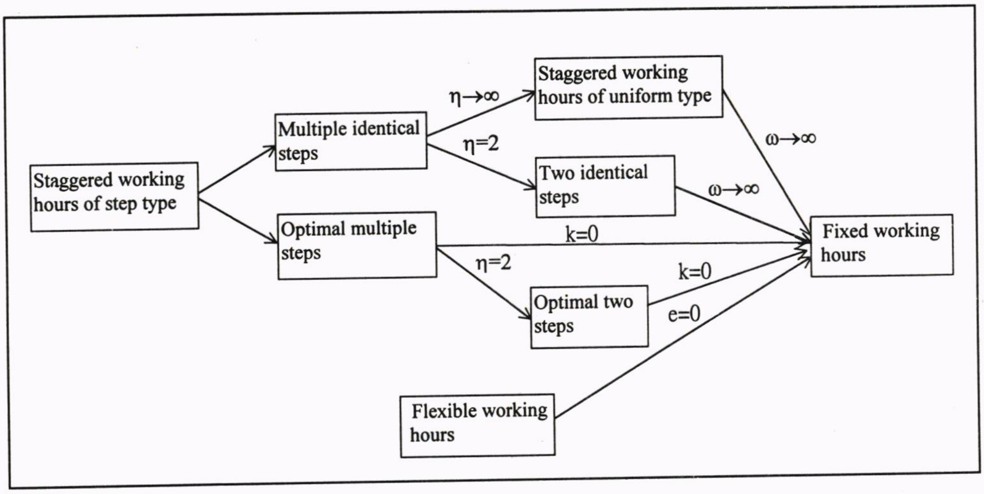


Figure 8. The relationship among alternative work schedules

5. SENSITIVITY ANALYSIS

We use the study of Small (1982) and Lan *et al.* (1998) with α=\$6.4/hr, β=\$3.9/hr and γ=\$15.21/hr. Assume that N=1800 commuters, each commuter drives a car and passes through a bottleneck to reach his/her destination. The capacity of the bottleneck is μ= 900 vehicles per hour. Under these conditions, the peak hour period is S = 2 hours. Under fixed working hours, all commuters start work at 9:00, under flexible and uniform staggered working hours between 8:30 and 9:00, and under staggered working hours of optimal two-step at 8:30 and 9:00. While applying our models, Table 3 summarizes those results. Commuters choose their departure times according to queue evolution as shown in Figure 9. The chart depicts the changes in peak traffic flow through the bottleneck at any moment throughout the morning rush hours, as well as the size of maximum queue length under various working hour systems.

Table 3. The results from examples

Items	Fixed working hours	Flexible working hours	Uniform staggered working hours	Optimal two step staggered working hours
Starting work hour	9:00	8:30~9:00	8:30~9:00	8:30；9:00
Departure time of first arrival	7:25	7:18	7:18	7:18
Departure time of in time arrivals	8:02	7:46~8:16	8:10	8:16
Departure time of last arrivals	9:25	9:18	9:18	9:18
Max. queueing delay (min.)	58	44	44	44
Travel cost (\$)	6.2	4.7	4.7	4.7
Total queueing delay (veh-hr.)	873	819	654	559
Max. queue length (veh.)	873	654	654	654

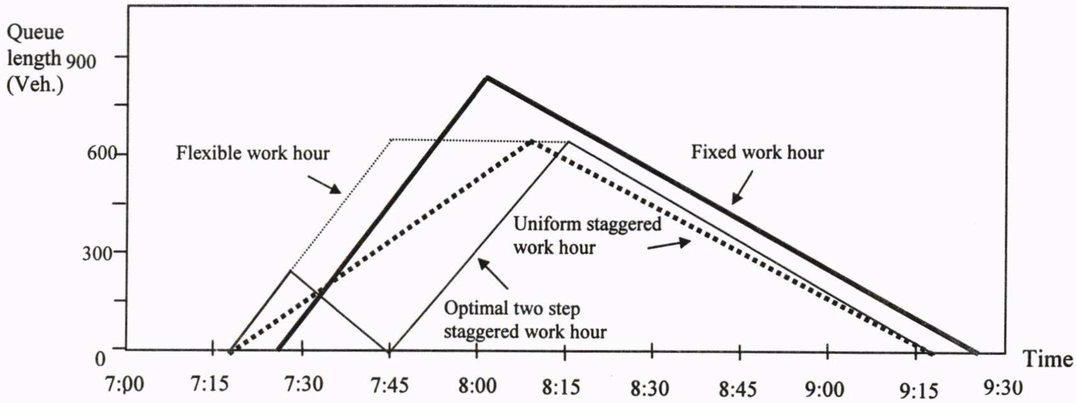


Figure 9. Time dependent queues under alternative work schedules

Data from the above example are used for sensitivity analysis to demonstrate how system parameters affect total queueing delay under alternative work schedules. Figure 10 summarizes the results of varying e, d and k. Under flexible working hours, $\partial W_f/\partial e = -(\beta/\alpha)[\gamma/(\beta+\gamma)]e\mu$, implying that as the flexible time length increases, the reduction in total queueing delay is larger. Under staggered working hours of uniform type, $\partial W_u/\partial d = -(1/2)(\beta/\alpha)[\gamma/(\beta+\gamma)]N$, which is independent of d, representing a situation in which total queueing delay is linearly proportional to staggered time length. When the length of flexible or staggered time is longer than the length of peak period, total queueing delay drops to zero. Under staggered working hours of optimal two steps, a situation in which $k<[\gamma/(\beta+2\gamma)]S$, $\partial W_s^*/\partial k = -(\beta/\alpha)[1/(\beta+\gamma)][\gamma N-(\beta+2\gamma)k\mu]$ suggests that as the staggered time length increases its effect on reducing total queueing delay becomes smaller. If $k > S/2$, the first and last group of commuters do not interact with each other; total queueing delay is constant and independent of k. When the length of flexible and staggered time are equal and less than half length of congestion period ($e=d=k<S/2$), total queueing delay ranks as $W_f>W_u>W_s^*$. When $e=k=S$, total queueing delay drops to zero.

Bottleneck capacity, μ , has an impact on the length of peak period, S , under a fixed number of commuters. Assume that the capacity is reduced because of road construction or increased due to widened, the bottleneck capacity varies from 600 vph to 1200 vph, with the duration of peak traffic flow from 3 hours to 1.5 hours, respectively. By setting the length of flexible/staggered working hours to 30 minutes, the effects on total queueing delay are summarized in Figure 11. Figure 11 ranks the total queueing delay as $W_p>W_f>W_u>W_s^*$.

Comparing the flexible and fixed work hour systems reveals that the difference of total queueing delay is small when the capacity is low because commuters have limited room to choose departure time under flexible working hours; however, the difference becomes larger when the capacity increases. In addition, comparing the uniform and staggered working hours of optimal two steps under the same length of staggered time reveals that the difference of total queueing delay is large when the capacity is low. This is attributed to that commuters should orderly pass through the bottleneck under uniform staggered working hours; however, the difference becomes smaller when the capacity increases. Total queueing delay of staggered working hours is lower than that of flexible working hours through out the range of capacity.

In general, rich commuters are less willing to spend time in traffic; in contrast, poor commuters are more willing to tolerate traffic congestion (Arnott *et al.*, 1989). Figure 12 summarizes the effects of changes in the ratio of unit cost of schedule delay to queueing delay (β/α) on total queueing delay, in which all other variables remained constant. Obviously, total queueing delay rises in direct proportion to the value of (β/α). Figure 13 depicts the sensitivity of total queueing delay to variations in he ratio of unit cost of schedule delay to lateness (β/γ) when all other variables are held constant. This figure clearly indicates that total queueing delay and their difference under alternative work schedules decrease as (β/γ) increases. Therefore, a situation in which the unit cost of lateness reduces will help resolve traffic congestion. In addition, the difference of total queueing delay among alternative work schedules would be smaller.

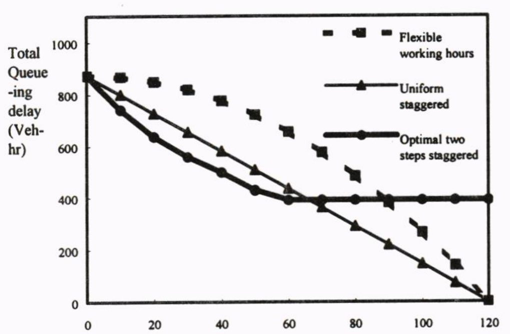


Figure 10. Flexible and staggered time length on total queueing delay

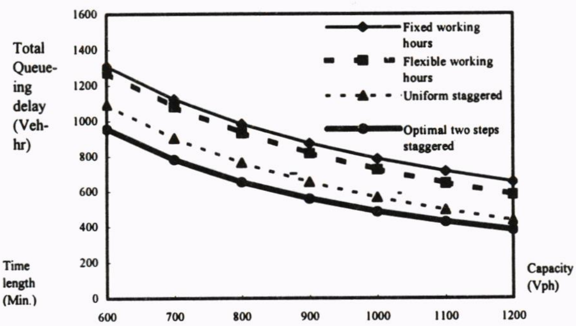


Figure 11. Bottleneck capacity on total queueing delay

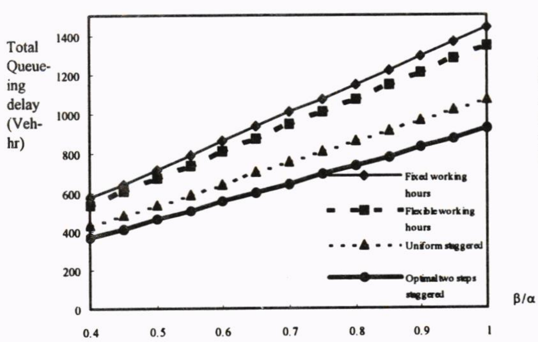


Figure 12. Unit cost of early schedule and queueing delay on total queueing delay

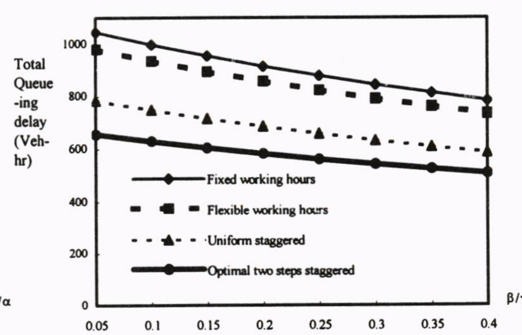


Figure 13. Unit cost of early and late schedule delay on total queueing delay

6. POLICY IMPLICATIONS AND CONCLUSIONS

Organizations worldwide have adopted flexible working hours for a few decades, resulting in positive benefits to companies, employees and sociality. Although some problems involving implementation in some companies still persist, it is not difficult to resolve (Nollen *et al.*, 1978). In staggered working hours system, employees once choosing the starting hours must abide by them for a certain period, thus, this system is less flexible.

While uniform staggered working hours are difficult to implement in practice, staggered working hours of step-type can be more easily implemented in a large company. A company determines some time slots of starting work, then assigns an appropriate number of employees to each slot or opens these slots chosen by employees. For instance, students and teachers of primary schools in Taipei, Taiwan start work at 7:30 am; employees and workers start work at 8:30 am; stock markets open at 9:00 am. Such measures effectively reduce traffic congestion. If one intends to introduce alternative work schedules to alleviate areawide traffic congestion, concentration of commuters' starting work times should be surveyed comprehensively.

Compared with fixed working hours, traffic delay reduced by the adoption of flexible working hours is in square proportion to the ratio of flexible time period to the length of peak period, $(e/S)^2$. The delay reduced by staggered working hours of uniform-type is linearly proportional to the ratio of staggered time length to the length of peak period, (d/S). The staggered working hours of step-type can yield even better performance, when the steps are optimized in such a way that the number of commuters in each step is managed at a rate equal to the staggered time multiplied by the saturation flow rate of the bottleneck.

Schedule delay is an important factor affecting the commuter's departure time. A reduction in the unit cost of early schedule delay (β) would be helpful in reducing traffic delays. Flexible working hours can reduce the commuter's schedule delay because commuters have no schedule delays if they arrive at work within a given length of flexible time, thereby reducing traffic congestion.

Flexible/staggered work schedules can reduce queueing delay. If the length of flexible or staggered time (e, k) exceeds the length of peak period (S), the traffic congestion can be completely removed. Under the same length of flexible and staggered time (e=k), although equilibrium travel costs of such two working hour systems are equal, total queueing delay for flexible working hours is higher than staggered ones. Total schedule delay for flexible working hours, however, is lower than staggered working hours. Although commuters may experience the same satisfaction with equal travel cost, staggered working hours should be considered as a priority strategy from the perspective of reducing traffic delays and social costs.

ACKNOWEDGMENTS

This research is granted by the ROC National Science Council under Contract No. NSC-2211-E-009-21.

REFERENCES

Arnott, R. J., de Palma, A. and Lindsey, R. (1989) Schedule delay and departure time decisions with heterogeneous commuters. **Transportation Research Record 1197**, 56-67.

Arnott, R. J., de Palma, A. and Lindsey, R. (1990a) Economics of a bottleneck. **Journal of Urban Economics 27**, 111-130.

Arnott, R. J., de Palma, A. and Lindsey, R. (1990b) Departure time and route choice for the morning commute. **Transportation Research 24B**, 209-228.

Arnott, R. J., de Palma, A. and Lindsey, R. (1993) A structural model of peak-period congestion : a traffic bottleneck with elastic demand. **American Economic Review 83**, 161-179.

Braid, R. M. (1989) Uniform versus peak-load pricing of a bottleneck with elastic demand. **Journal of Urban Economics 26**, 320-327.

Daganzo, C. F. (1985) The uniqueness of a time-dependent equilibrium distribution of arrivals at a single bottleneck. **Transportation Science 19**, 29-37.

de Palma, A. and Arnott, R. J. (1986) Usage-dependent peak-load pricing. **Economics Letters 20**, 101-105.

D'Este, G. (1985) The effect of staggered working hours on commuter trip durations. **Transportation Research 19A**, 109-117.

Ferguson, E. (1990) Transportation demand management: planning, development, and implementation. **Journal of the American Planning Association 56**, 442-456.

Henderson, J. V. (1981) The economics of staggered work hours. **Journal of Urban Economics 9**, 349-364.

Hendrickson, C. and Kocur, G. (1981) Schedule delay and departure time decisions in a deterministic model. **Transportation Science 15**, 62-77.

Jones, D., Nakamoto, T. and Cilliers, M. (1978) **Transportation System Management Actions : Implications of Flexible Work Hours.** US DOT Report.

Jovanis, P. P. (1981) Flexible work hours and mode change: interpretation of empirical findings from San Francisco. **Transportation Research Record 816**, 11-18.

Kuwahara, M. (1990) Equilibrium queueing patterns at a two-tandem bottleneck during the morning peak. **Transportation Science 24**, 217 -229.

Laih, C. H. (1994) Queueing at a bottleneck with single and multi-step tolls. **Transportation Research 28A**, 197-208.

Lan, L. W. and Chang, S. H. (1987) Design and analysis of congestion toll. **Traffic and Transportation 9**, 75 -87 (in Chinese).

Lan, L. W. and Chen, T. T. (1998) The effect of alternative work schedules on commuter trip delays. **Transportation Planning Journal 27**, 185-212 (in Chinese).

Newell, G. F. (1987) The morning commuters for nonidentical travelers. **Transportation Science 21**, 74-88.

Nollen, S. and Martin, V. (1978) **Alternative Work Schedules; Part I : Flextime.** American Management Association.

O'Malley, B. M. (1975) **Work Schedule Change to Reduce Peak Transportation Demand**. Transportation Research Board, Special Report **153**.

Small, K. A. (1982) The scheduling of consumer activities: work trips. **American Economic Review 72**, 467-479.

Smith, M. J. (1984) The existence of a time-dependent equilibrium distribution of arrivals at a single bottleneck. **Transportation Science 18**, 385-394.

Tabuchi, T. (1993) Bottleneck Congestion and Modal Split. **Journal of Urban Economics 34**, 414-431.

Transportation Research Board (1980) **Alternative Work Schedules: Impacts on Transportation.** NCHRP SYN **73**.

Vickrey, W. S. (1969) Congestion theory and transport investment. **American Economic Review 59**, 251-260.