A BENEFIT ASSIGNMENT METHOD GENERATED THROUGH A SOIL TRANSPORT MODEL

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Abstract: In growing metropolises, many projects are under construction during the same or overlapping time periods. Some soil transport models have been developed to co-ordinate construction sites and recycle the surplus soil under the condition that the whole construction sites join the coalition for recycling. To maintain the whole coalition, the benefit generated by the coalition should be fairly distributed to each project. In this paper, the benefit assignment method is developed, applying the nucleolus of cooperative game theory. Then the calculation procedure of the "nucleolus" is described. Finally, the method applies to a hypothetical case study to prove the availability of the method.

1. INTRODUCTION

In metropolises, many projects are under construction during the same time period. At project sites, a large amount of soil is gathered from supplier sites and abolished to disposal sites for construction work. It is necessary to recycle these soil among construction sites in order to decrease the soil transport cost and lessen the environmental damages due to cutting hills, soil disposal to valleys/sea, and lorry exhausted gas accompanied with soil transport etc.

Some soil transport models (for example, Minami and Shimazu, 1988, Wada and Yamamoto, 1993; Tomita and Terashima, 1996; Tomita and Hayashi et al., 1997) are developed to co-ordinate construction sites. These models are useful under condition that the whole construction sites join to the coalition for recycling. However, in order to form and maintain the whole coalition, the benefit generated through soil transport models should be fairly distributed to each project.

In this paper, the assignment method of the benefit is developed, applying the "nucleolus" of cooperative game theory. The nucleolus means the benefit assignment to minimize the maximum dissatisfaction of coalitions, which means that the nucleolus is the most stable benefit assignments to prevent the coalition from collapsing.

In the next section, a soil transport model of Tomita and Terashima (1996) is explained briefly. In section three, basic concept of the benefit assignment method is presented by applying cooperative game theory. According to the theory, it is known that the most fairly and stable benefit assignment is "nucleolus". Then the nucleolus calculation procedure is explained, followed the definitions of players, characteristic function and dissatisfaction function of the soil recycling game. In section four, the method is applied to a hypothetical case study to prove the availability of the method.

2. OUTLINE OF SOIL TRANSPORTATION MODEL

A soil transport model of Tomita and Terashima (1996) is formulated as a linear programming model. The objective function is the total soil cost including 1) soil transport costs between project sites generating/ requiring soil, soil storage yards, soil quality improvement plants, soil supplier/disposal sites, 2) soil quality improvement cost, 3) soil storage cost, 4) soil purchase cost, and 5) soil disposal cost. The soil flow pattern between project sites and facilities is shown in Figure 1.

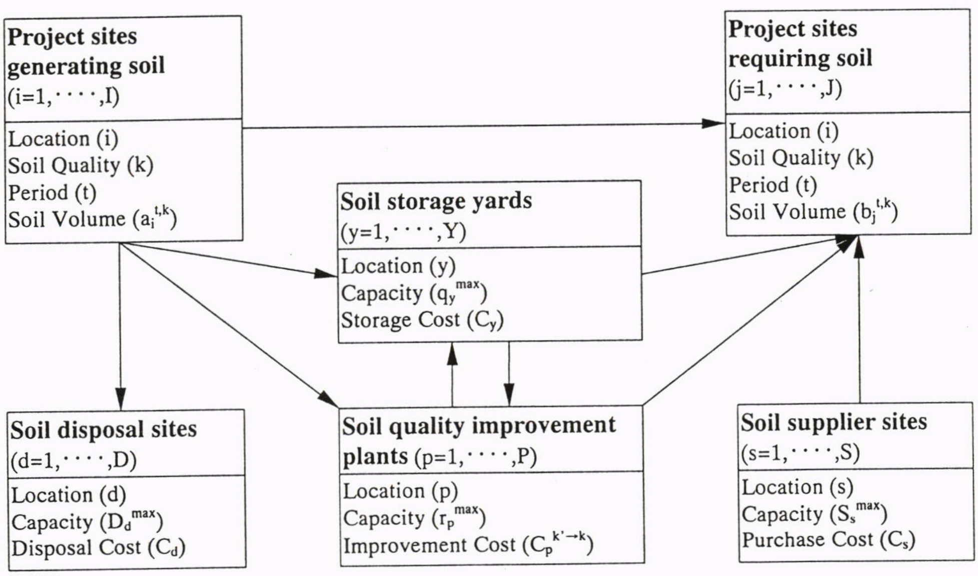


Figure 1. Soil Flow between Project Sites and Facilities

The conditions in the model are described as follows: 1) balances between inflow and outflow volumes of soil at each facility and each project, and 2) capacities of soil facilities such as soil quality improvement plants, soil storage yards, soil supplier sites and soil disposal sites.

The above objective function and conditions are formulated in Tomita and Terashima (1996) and Tomita and Hayashi et al.(1997).

3. BENEFIT ASSIGNMENT METHOD GENERATED THROUGH SOIL TRANSPORT MODEL

3.1 Concept of the Method and the Related Terms of Game Theory

Soil recycling can be seen as a cooperative game when projects are assumed as players. In this case, in order to realize the minimization of the total soil cost using the model in section 2, it is necessary to form the cooperative relationship in the whole projects. The relationship is termed as the **"whole coalition"** in cooperative game theory and the projects in the coalition behave in the way to attain the aim of the coalition.

According to the game theory, two conditions are required in order to form the whole coalition. One condition is that a benefit function of coalition is **"superadditive"**, which is defined as $\nu(S_1 \cup S_2) \geqq \nu(S_1) + \nu(S_2)$, for all coalition S_1, S_2, (where $\nu(S)$ is a benefit function of coalition S). This means larger coalition can generate larger benefit.

Another condition is that the benefit of the whole coalition is allocated fairly so as to satisfy **"coalitional rationality"**. To allocate the benefit fairly to players is termed as **"side payment"**. Then the coalitional rationality is defined as that the benefit of any coalition S within the whole coalition doesn't exceed over total benefit allocated the whole coalition for players within the coalition: $\sum_{i \in S} X_i \geq \upsilon(S)$ (where $\upsilon(N) = \sum_{i \in N} X_i$, $X_i \geq 0$ and X_i is the allocated benefit for project i). A set of benefit allocation satisfying the coalitional rationality is defined as **"core"** if the game is superadditive. Then one component of the "core" is **"nucleolus"**, which is defined as a benefit allocation to minimize the maximum dissatisfaction among those of all coalitions. In this sense, the nucleolus can be considered to be the most stable benefit allocation to maintain the whole coalition.

In the following section, the **"benefit/dissatisfaction functions"** are formulated in order to define the soil recycling game as a cooperative game, and it is clarified that the benefit function is superadditive. Then in section 3.3 the nucleolus calculation method is described.

3.2 Definitions of Benefit/Dissatisfaction Functions

In the soil recycling game, the benefit function $\nu(S)$ can be defined as the maximum benefit caused by the coalition S. The benefit can be thought as the difference between two costs in equation(1): one is the minimized cost through the formation of the coalition S, that is C_S, and another is the total cost of projects in the case without coalition S, that is $\sum_{i \in S} C_{\{i\}}$. In the case without coalition S, each project behaves alone as a single coalition.

$$\nu(S) = \sum_{i \in S} C_{\{i\}} - C_S \qquad \text{---------------- (1)}$$

where $\nu(S)$: Benefit of coalition S
$C_{\{i\}}$: Cost of project (i) who behaves alone as a single coalition
C_s : Minimum cost of coalition S, which can be calculated through the soil transport model in section 2

This benefit function $\nu(S)$ is superadditive when $\nu(S_1 \cup S_2) \geqq \nu(S_1) + \nu(S_2)$, for any coalitions S_1, S_2. The condition can be satisfied in the soil recycling game due to the following reasons. The first term on the right side of equation (1), that is $\sum_{i \in S} C_{\{i\}}$, is not different in the either case of $\nu(S_1 \cup S_2)$ or $\nu(S_1) + \nu(S_2)$; so the difference is only the second term of equation (1); the cost $C_{(S_1+S_2)}$ of coalition $S_1 \cup S_2$ is less than or equal to the total cost $C_{S_1}+C_{S_2}$ of each coalitions S_1, S_2 due to the definition of C_s.

Then for the calculation of nucleolus, the dissatisfaction function e(S,X) of coalition S against the benefit allocation X is required. When the benefit of the whole coalition $\upsilon(N)$ is allocated for projects by $X=(X_1,X_2,----,X_i,---,X_n)$ (where $\upsilon(N) = \sum_{i \in N} X_i$, $X_i \geq 0$ and X_i is the allocated benefit for project (i)), the dissatisfaction function e(S,X) can be defined as the difference between the benefit of coalition S and the total allocated benefit of whole coalition for projects within the coalition S:

$$e(X,S) = \nu(S) - \sum_{i \in S} X_i \qquad \text{---------------- (2)}$$

As it is clarified above that the benefit function $\nu(S)$ is supereadditive, so the whole coalition can be formed if the benefit of the whole coalition is allocated for each project by nucleolus. The nucleolus can be calculated through the procedure described in the next section.

3.3 Nucleolus Calculation Procedure

The nucleolus of the soil recycling game can be calculated by applying the procedure developed by Machler and Peleg et al. (1979). In the procedure, linear programming problems are solved successively until the nucleolus is identified, following the steps below.

Step 1: Identification of benefit function ν (S)

Total soil costs C_s of all coalitions S including single coalition whose member is only a single project, are calculated through the soil transport model of section 2. By using the values, the benefit function ν (S) can be identified.

Step 2: Minimizing maximum dissatisfaction e(X,S)

To minimize the dissatisfaction e(X,S) defined in section 3.2, the following linear programming problem is solved. The minimized value z is set as z^1.

$$z \rightarrow \min \text{ subject to } \nu(S) - \sum_{i \in S} X_i \leq z, \nu(N) - \sum_{i \in N} X_i = 0, X_i \geq 0 \qquad \text{---------------- (3)}$$

The solution is not a single solution but a solution set. The solution set is called "core" which is the benefit allocation set which minimizes the greatest dissatisfaction among those of all coalitions.

Step 3: Minimizing the second largest dissatisfaction e(X,S)

Within the core set in Step 2, various benefit allocations are included. To minimize the second largest dissatisfaction, the following linear programming is solved and the core set of Step 2 is diminished.

$$z \rightarrow \min \text{ subject to } \upsilon(S) - \sum_{i \in S} X_i = z^1, \ \nu(S) - \sum_{i \in S} X_i \leq z,$$

$$\nu(N) - \sum_{i \in N} X_i = 0, \ X_i \geq 0 \qquad \text{---------------- (4)}$$

where z^1: the minimized value of objective function of the problem (3)

The above linear programming problem is induced by replacing some inequalities with equalities, which are the result of having solved the problem (3).

Step 4 Iteration until nucleolus is identified

The procedure similar to Step 2 and Step 3 is continued successively until the core set is diminished to a single component $X=(X_1,X_2,----,X_i,---,X_n)$, which is nucleolus.

4. APPLICATION

4.1 Conditions for Application

To prove the availability of the method in the previous section, it is applied under the following conditions:

1) Locations of project sites and soil recycling facilities:

The locations of project sites and soil recycling facilities are assumed as shown in Figure 2, which includes 8 project sites, a soil quality improvement plant, a soil storage yard, a soil supplier site and a soil disposal site. In the figure, some project sites have both soil gathering/requiring sites.

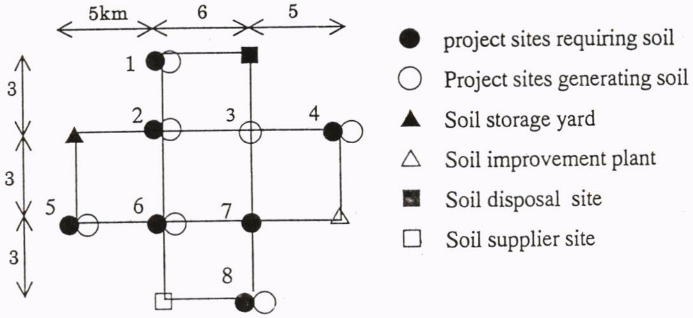


Figure 2. Location of Project Sites and Soil Recycling Facilities

2) Soil conditions of project sites:

Soil conditions, including soil volume, soil quality, project commencing time and project period, are assumed as shown in Table 1.

Table 1. Soil Conditions of Construction Work

No. of project site	Construction works generating soil				Construction works requiring soil			
	Soil volume (m³)	Soil quality (*)	Commencing time	Construction period	Soil volume (m³)	Soil quality (*)	Commencing time	Construction period
1	2,000	1	1	4	1,800	1	3	3
2	1,000 500	2 1	4 3	1 1	1,000 -	1 -	5 -	1 -
3	-	-	-	-	3,000	1	2	4
4	5,000	1	2	4	3,600	2	3	4
5	2,300	2	1	3	1,500	1	1	2
6	1,200	1	5	2	1,200	1	4	2
7	3,000	1	2	3	-	-	-	-
8	15,000 8,000	2 1	1 3	6 4	20,000 -	1 -	4 -	2 -
Total	38,000	-	-	-	32,100	-	-	-

(*) Values (1),(2) mean high/low qualities respectively

3) Charges of facility use and soil transport
Unit charges of facilities usage and soil transport are assumed as shown in Table 2.

Table 2. Charges/Capacities of Facility Use and Soil Transport Charge

Storage yard	Capacity	(Q_y^{max})	10,000 (m^3/ period)
	Storage charge	(C_y)	1,500 (m^3/ period)
Soil quality improvement plant	Capacity	(R_p^{max})	8,000 (m^3/ period)
	Improvement charge	(C_p)	5,000 (yen / m^3·period)
Supplier site	Capacity	(S_s^{max})	20,000 (m^3)
	Purchase charge	(C_s)	4,000 (yen / m^3)
Disposal site	Capacity	(D_d^{max})	20,000 (m^3)
	Disposal charge	(C_d)	2,000 (yen / m^3)
Soil Transport charge			100 (yen / m^3·km)

4) Periods and Soil Quality Levels
It is assumed that projects are implemented for 6 periods and there are two levels of soil quality: level 1 is higher than level 2 and the level 2 can be improved to the level 1 through a soil quality improvement plant.

4.2 Application Results

(1) Soil flow, soil cost and the environment effect

Soil flow to minimize the total soil cost after forming the whole coalition is shown in Figure 3, which means most volume of soil is recycled.

Figure 4 shows 1) the cost of the above soil flow and, for comparison, 2) the cost in the case of "single coalition" where each project behaves alone, doesn't recycle and only purchase/dispose the soil. Comparing the above cases, the cost of whole coalition is around 40% smaller than the single coalition case. The saving is the benefit generated by the whole coalition.

The transport, purchase and disposal costs decrease by 20%, 80% and 40% respectively. This means that the corresponding soil volumes also decrease. As a result, the environmental damages due to cutting hills, soil disposal to valleys/sea, and lorry exhausted gas accompanied with soil transport would be lessened similarly.

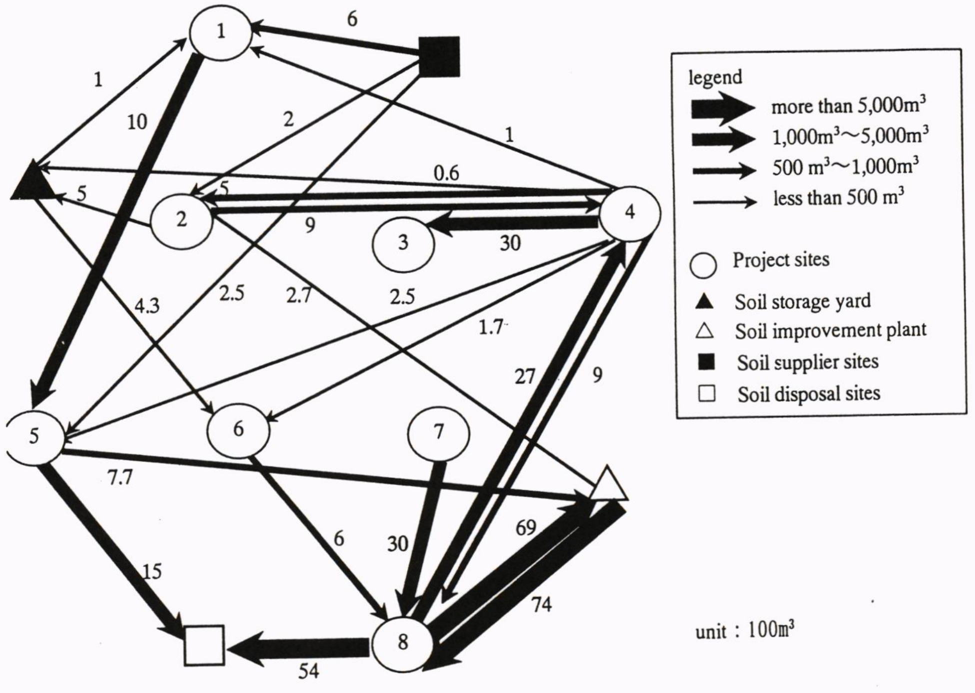


Figure 3. Soil Flow in Case of Whole Coalition

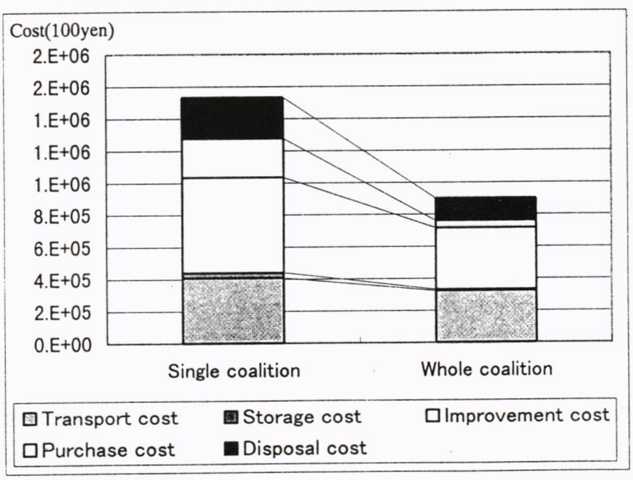
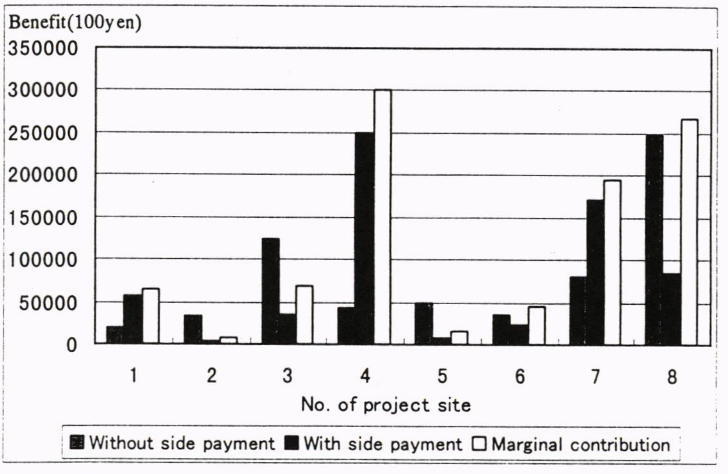


Figure 4. Total Soil Costs in Cases of Single/Whole Coalitions

(2) Nucleolus and costs of each project

The total benefit of the whole coalition is allocated as side payment to each project by nucleolus measured by the method developed in section 3. Figure 5 shows 1) benefit allocation in the case with side payment, whose value is nucleolus, and in addition, 2) benefit allocation in the case without side payment, and 3) marginal contribution to the whole coalition of each project. Comparing the cases with/without side payment, for example, the benefits allocated to project 4 and 7 in the case with side payment is much larger than in the case without side payment. It is understood by the difference of the marginal contribution displayed as the right bar in Figure 5.



(Note) Marginal contribution means marginal contribution to the whole coalition.

Figure 5. Benefit Allocation in Cases with/without Side Payment

However the allocated benefit of project 8 is an exception. The benefit in the case with side payment is much smaller than that without side payment, even though the marginal contribution is high. The reason can be thought as follows: the soil volume of project 8 is much larger than the other soil volumes of projects, therefore the marginal contribution is larger than the expected one; on the other hand, the allocated benefit of project 8 is smaller, because the dissatisfaction of project 8 doesn't depend on whether project 8 joins a coalition or not.

Figure 6 shows the costs in the cases with/without side payment, and is added for comparison the cost in the case of single coalition. Comparing them, the costs with/without side payment are smaller than the costs of single coalition. It is thought that the costs in the case without side payment are smaller occasionally; on the other hand, the costs in the case with side payment never exceed inevitably, because the allocated nucleolus in the case with side payment satisfies individual rationality.

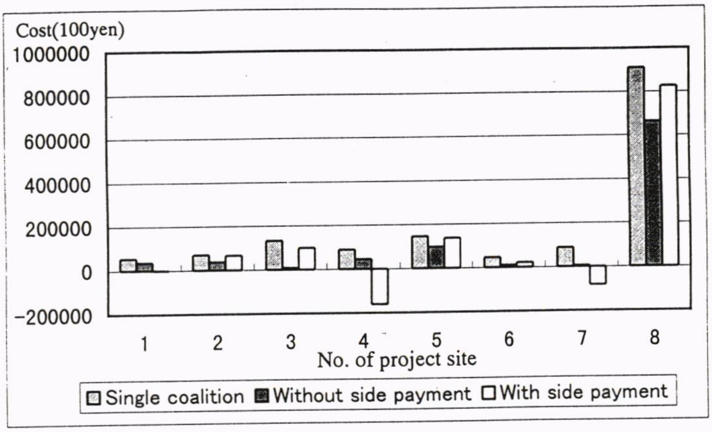


Figure 6. Total Soil Costs in Cases of Single Coalition and with/without Side Payment

(3) Dissatisfaction of Coalitions

Figure 7 shows the dissatisfaction of coalitions in falling order in the cases of "with/without side payment". If the side payment is not implemented, some coalitions have great dissatisfactions in forming the whole coalition.

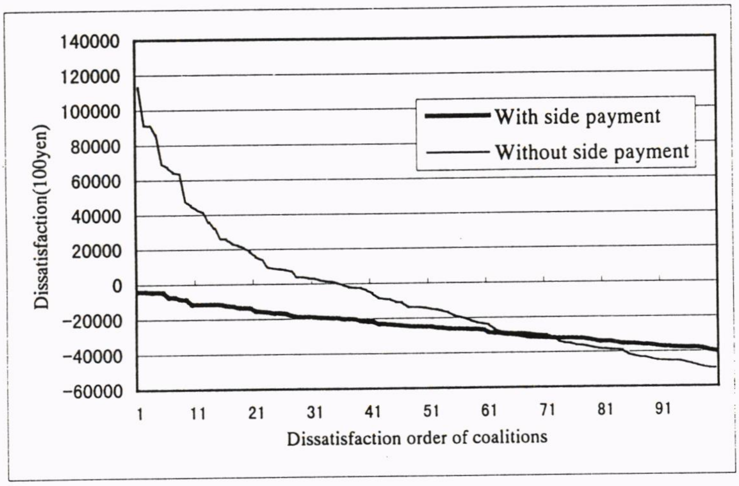


Figure 7. Dissatisfaction Order of Coalitions in Cases with/without Side Payment

However, if the side payment is done by the nucleolus, the large dissatisfactions are minimized to be the negative value. This clarifies the effectiveness of the benefit allocation by nucleolus.

5. CONCLUSION

In this paper, a benefit assignment method generated through a soil transport model was developed, then the availability was confirmed in the hypothetical case study. By using both this method and some existing soil transport models, it is expected that the surplus soil recycling would be promoted and the environmental damages would lessen.

However, the number of projects dealt with in the method is limited due to the calculation load of nucleolus, so the development of an efficient calculation method is hoped in the future.

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