AN ITERATIVE METHOD TO ANALYZE INTERACTIVE EFFECTS OF LONG-TERM TRANSPORTATION AND HOUSING POLICIES

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abstract : This study developed a model which can analyze housing polices and/or transportation policies. The model can explicitly consider the ripple interactive effects between housing market and transportation system. For the simplicity and applicability to real world, this model focuses on only the interaction between transportation system and housing market with fixed other land use allocations. This model has three sub-models ; residential location choice sub-model, housing market equilibrium sub-model and transportation demand analysis sub-model. This study also developed a solution algorithm to solve the equilibrium problem between transportation and housing market. An iterative method was applied to solve this equilibrium problem between two systems : transportation and housing market system.

1. INTRODUCTION

In Seoul metropolitan area, South Korea, several huge-scale new towns such as Bundang, Pyungchon, Sanbon and Ilsan near Seoul city were planned and built as bed-towns of Seoul. Many places of old low-story apartment sites (usually, 5 stories) inside of Seoul city were considered to be redeveloped as high-rise apartment sites (more than 20 stories). Beside of such housing development and redevelopment, huge-scale of highway and subway systems are also being continuously constructed in Seoul metropolitan area such as the inner and outer beltway and the second stage of heavy-rail subway systems. Many scholars emphasized the importance of considering interactive effects between land use and transportation systems in large-scale and long-term transportation and land-use plans. However, such huge housing projects and transportation projects were not systemically analyzed to see their interactive effects on each other system as the long-term equilibrium states. They were usually analyzed independently to see effects on its own system rather than interactive effects between two systems.

This paper will introduce a practical method which can analyze not only the effects of large-scale housing policies on transportation system, but also the effects of large-scale transportation planning on housing market in addition of the direct effects on their own systems. This transportation and housing interactive model was developed to analyze the interactive effects between transportation and housing policies by simplifying the theoretical LUTEM model (Land Use and Transportation Equilibrium Model) developed by Kim (1995). For the simplicity and applicability to real world, this model focuses on only the interaction between transportation system and housing market with fixed other land use allocations. This simplified LUTEM model was composed with residential location choice model with given number of each type of houses at each zone, housing market equilibrium model between demand and supply of houses, transportation demand analysis model and equilibrium model between transportation system and housing market. This model can be applied to analyze housing policies or transportation policies while the interaction effects between transportation and housing systems are explicitly reflected into the model.

2. LITERATURE REVIEW

Alonso (1964) established the foundations of a mathematical model of urban land use without considering transportation network congestion in monocentric city. After this pioneering research, scholars in various fields have paid attention to the economic theory of urban land use and transportation. The author, Kim (1994), categorized transportation and land use models into three types. The models analyzed in Bly and Webster (1987) and Mackett (1993) can be also classified into these three type of models. The first type is Mills-type model. One of important achievement in this research field was the introduction of linear programming to general equilibrium model with transportation network congestion accomplished by Mills (1972). This model has been extended by Hartwick and Hartwick (1974), Kim, T. J. (1979) and Moore (1986) to incorporate intermediate goods, multiple centers, multiple travel modes and perfect foresight dynamics. This type of land-use and transportation interaction model was called Mills-type model in Kim (1994). The Mills-type models usually focused on optimizing system by minimizing costs rather than reflecting choice behavior into the model. This type of models has strong theoretical ground, but it is not easy to calculate the solution of the model if there are some nonlinear function in the mathematical formulation. Therefore, it is difficult to apply this type of models to analysis of real transportation and land use policies.

A second type of land use and transportation interaction models stemmed from Lowry 's model (1964) which considered the cyclical effects on local employment and housing increments each other by initial basic employment growth. The Lowry's model did not reflect traffic congestion effects because it assumed fixed transportation costs between two zones. Putman (1983) greatly enhanced the Lowry's model by introducing the concept of Wilson's entropy or gravity model in allocation of employees and residents and by considering traffic congestion in transportation network. These models are called Lowry-type model. These Lowry-type models physically distribute activities over spaces rather than optimizing system or reflecting individual choice behavior in industrial and residential location choice. The computation of Lowry-type models is relatively easy. Therefore, this type of models can be easily applied to huge-scale of real transportation and housing problems. However, these models have theoretical shortcomings in allocating activities. The equations of distributing activities in the Lowry-type models do not admit a clear behavioral interpretation.

The third type of transportation and land use interaction models was called a stochastic equilibrium model of transportation and land use. This type of models tried to reflect imperfect information, effects of variables not included in models, inconsistent behavior, etc. into random variables. Therefore, these models distribute activities over spaces in terms of probability of residential or industrial location choice. Kim (1990) and Anas and Kim (1996) developed a theoretical model of this type. Kim (1995) tried to develop a practical model (LUTEM : Land Use and Transportation Equilibrium Model) by simplified some assumptions while minimizing losses of theoretical soundness. This paper will suggest a model which simplify the LUTEM for more practicability in policy analysis by only focusing transportation and housing market systems while assuming allocations of all industrial activities are given as exogenous variables.

3. MODEL STRUCTURE

The simplified model of LUTEM was consisted of three major sub-models : residential location choice sub-model, housing market equilibrium sub-model and transportation demand analysis sub-model.

3.1. Residential location choice and housing market equilibrium sub-model

The basic concept of the residential location choice sub-model and housing market equilibrium sub-model were similar to CATLAS (the Chicago Area Transportation/Land

Use Analysis System) model developed by Anas (1983). The CATLAS is a dynamic housing model with fixed transportation system and travel time. Even though the residential location choice sub-model and housing market equilibrium sub-model used in this study are conceptually similar to the CATLAS in a view of theoretical base, it is completely different in model structure and model specification. The sub-models developed in this research are more simplified static models to analyze long-term housing policy in real world. The residential sub-model used a nested logit model that is constructed by housing type choice at each zone (lower level choice) and zonal choice (upper level choice). The utility function in the nested logit model at lower level is composed with housing rent and amount of housing space for each type of houses. The utility function at upper level of nested logit model is composed with neighborhood composition in aspect of housing quality, amount of public facilities such as park, road etc., and inclusive values (expected maximum utility) from lower nest. The upper level of the nested logit model is to represent choice behavior of a household to choose a residential zone.

The model for housing type choice at a given zone, which is the lower-level choice in nested logit model structure, can be explained as followings;

$$P_{h|ig} = \frac{\exp(V_{h|ig})}{\sum_{H} \exp(V_{H|ig})} \qquad \text{(Eq. 1)}$$

$$V_{H|ig} = f\left(r_{H|j}, q_j, Y_g\right)$$

where $P_{h|ig}$ = probability of choosing house type h with given residential zone i and given decision maker in income group g

$V_{H|ig}$ = utility function of house type H for a decision maker who resides in zone i and who is classified as income group g

$r_{H|i}$ = rent of house type H at zone i

q_H = amount of space of house type H

Y_g = average income of resident classified as income group g

The utility function $V_{H|i}$ can be any form of function, linear form or nonlinear form. However, for simplicity in estimation and interpretation, linear utility functions are usually adapted in practical application. For simplicity of the model, following utility function is suggested in this research ;

$$V_{H|ig} = C_H + \alpha \ln(Y_g - r_{H|i}) + \beta \ln q_H \qquad \text{(Eq. 2)}$$

where C_H = alternative specific constant for house type H

α , β = parameters

The model for residential zone choice, which is the upper-level choice in nested logit model structure, can be explained as followings ;

$$P_{i|jg} = \frac{\exp[W_{i|g} + (1-\sigma) I_{i|g}]}{\sum_J \exp[W_{J|g} + (1-\sigma) I_{J|g}]} \quad \text{(Eq. 3)}$$

$$W_{J|g} = g(B_J, G_J, T_J)$$

where $P_{i|jg}$ = probability of choosing zone i as the residential location for a employee who works at zone j and is in income group g

σ = coefficient for inclusive value which is the measure of correlation due to dwelling similarity within a zone.

$I_{J|g}$ = inclusive value of zone J for a person in income group g

$W_{J|g}$ = utility function of zone J as a residential location for a person in income group g

B_J = ratio of high quality houses to total number of houses at zone J

G_J = ratio of public facility area (park, school, etc.) to total area at zone J

T_J = transportation cost from resident's job location to zone J

The inclusive value $I_{J|g}$ can be calculated as following ;

$$I_{J|g} = \ln\left[\sum_H \exp(V_{H|Jg})\right] \quad \text{(Eq. 4)}$$

The utility function W_J can be also any form of function, linear form or nonlinear form. This research suggests the following utility function of zone J;

$$W_{J|g} = \gamma B_J + \delta G_J + \lambda \frac{T_J}{Y_g} \quad \text{(Eq. 5)}$$

where γ , δ , λ = parameters

Therefore, the demand functions for a specific type of housing k at zone i can be expressed as followings;

$$D_{hi}(\overline{R}) = \sum_{j}\sum_{g} E_{jg} P_{i\mid jg} P_{h\mid ig} \qquad \text{(Eq. 6)}$$

where $D_{hi}(\overline{R})$ = demand of housing type h at zone i as function of rent vector

E_{jg} = number of employees in income group g at industrial zone j, who are head of the household

The housing market clear is performed with the housing market equilibrium sub-model by adjusting the housing rents for each type at each zone. The functional form of the housing market equilibrium sub-model is as following ;

$$\sum_{j}\sum_{g} E_{jg} P_{i\mid jg} P_{h\mid ig} = S_{hi}, \qquad \forall\ i, h \qquad \text{(Eq. 7)}$$

where S_{hi} = number of house type h in zone i (fixed supply of house type h in zone i)

Anas (1982) proved the existence and uniqueness of solution for stochastic walrasian equilibrium if demand function satisfies the conditions $\partial D_{hi}(\bar{r}) / \partial r_{h\mid i} \prec 0$ and $\partial D_{hi}(r) / \partial r_{h\mid j} \succ 0$. The (Eq. 7) has same functional form and satisfies the above conditions. Therefore, we can solve (Eq. 7) as function of endogenous variable rents with given transportation costs, if there is a efficient algorithm. The equation (Eq. 7) is a simultaneous nonlinear equations which can be solved by some modified Newton-Raphson Methods. This housing market equilibrium sub-model will find the stochastic equilibrium housing rents of each type at each zone with given transportation system and travel time of each transportation mode.

3.2. Transportation demand analysis sub-model

The transportation demand analysis sub-model used in this study is same as the traditional four step transportation demand estimation process. However, it is conceptually different from the traditional one at the steps of trip generation and trip distribution analysis in case of work trips. We can find out the origin-destination work trips directly from the residential location choice model and housing market equilibrium model. Therefore, we don't have to analyze trip generation and trip distribution for work purpose trips in this model. This P-A trips (production-attraction trips) for work can be explained by following equation ;

$$PA_{ij}^{w} = 2\left(\sum_{g} E_{jg} P_{i\mid j}\right) \qquad \text{(Eq. 8)}$$

where PA_{ij}^{w} = number of trips of which production zone is i and attraction zone is j for work

For other purpose of trips such as shopping trip, school trip, business trip, recreational trip and so on, the category analysis for trip generation and gravity model for trip distribution can are applied in this study. Trip generation model for these purpose trips except work trips can be explained as following equations ;

$$P_i^a = \sum_c H_{ci} t_c^a \qquad \forall\ a \qquad \text{(Eq. 9)}$$

$$H_{ci} = \sum_j \sum_g f_{gc}\left(E_{jg} P_{i|j}\right) \qquad \forall\ c, i \qquad \text{(Eq. 10)}$$

$$a_j^a = \sum_b E_{bj} t_b^a \qquad \forall\ a \qquad \text{(Eq. 11)}$$

where P_i^a = number of trip production at zone i for trip purpose a

t_c^a = production trip rate of a household in household type c for trip purpose a

H_{ci} = number of household in household type c at zone i

f_{gc} = proportion of household type c in employee income group g

a_j^a = number of trip attraction at zone j for trip purpose a

E_{bj} = number of employee in industry type b at zone j

t_b^a = attraction trip rate of an employee in industry type b for trip purpose a

Total production trips and total attraction trips should be equal as a whole system, but (eq. 9) and (eq. 11) can be produced different total number of trips for each purpose of trips. Therefore, an analyst may need to balance the attraction trips based on production trips as following;

$$A_j^a = a_j^a \frac{\sum_i P_i^a}{\sum_j a_j^a} \qquad \text{(Eq. 12)}$$

where A_j^a = finally balanced trip attraction at zone j for trip purpose a

Except work trips, produced trips and attracted trips for each purpose at each zone by trip generation analysis should be distributed over space by a trip distribution model. In this study, the gravity model is used such as following equations;

$$PA_{ij}^{a} = P_i \frac{A_j^a F_{ij}^a (C_{ij})}{\sum_J A_J^a F_{iJ}^a (C_{iJ})} \qquad \text{(Eq. 13)}$$

$$F_{ij}^{a} (C_{ij}) = \rho_1^a C_{ij}^{\rho_2^a} \exp\left(C_{ij}^{\rho_3^a}\right) \qquad \text{(Eq. 14)}$$

where $F_{ij}^{a} (C_{ij})$ = friction factor or impedance function from zone i to zone j for trip purpose a

C_{ij} = generalized transportation cost from zone i to zone j which implicitly implies travel time, travel cost and so on

ρ_1^a , ρ_2^a , ρ_3^a = parameters in impedance function for trip purpose a

As mentioned above, trip distribution for work trips can be calculated by using (eq. 8) of which input variables are the results of residential location choice sub-model and housing market equilibrium sub-model.

For the mode choice analysis, this study uses the multinomial logit model. The functional form of this model is same as (eq. 1). However, the form of the utility function and independent variable used in the utility function are different from (eq. 1). The mode choice model used in this study are as followings;

$$P_{m|ij}^{a} = \frac{\exp(V_{m|ij}^{a})}{\sum_M \exp(V_{M|ij}^{a})} \qquad \text{(Eq. 15)}$$

$$V_{m|ij}^{a} = C_m^a + \beta_1^a COST_{m|ij} + \beta_2^a IVTT_{m|ij} + \beta_3^a OVTT_{m|ij} \qquad \text{(Eq. 16)}$$

where $P_{m|ij}^{a}$ = probability of choosing mode m between zone i and zone j for trip purpose a

$V_{m|ij}^{a}$ = utility function of mode m if a decision maker travels between zone i and zone j for trip purpose a

C_m^a = alternative specific constant for transportation mode m for trip purpose a

$COST_{m|ij}$ = travel cost of mode m between zone i and zone j

$IVTT_{m|ij}$ = in-vehicle travel time of mode m between zone i and zone j

$OVTT_{m|ij}$ = out-of-vehicle travel time of mode m between zone i and zone j

β_1^a , β_2^a , β_3^a = parameters in utility function for trip purpose a

By using (Eq. 8), (Eq. 13) and (Eq. 15), we can calculate the P-A trips by trip purposes and transportation modes as following;

$$PA^{a}_{mij} = PA^{a}_{ij} P^{a}_{m|ij} \quad \forall \text{ a, m, i, j} \qquad \text{(Eq. 17)}$$

where PA^{a}_{mij} = number of P-A trips of which production zone is i and attraction zone is j by using transportation mode m for trip purpose a including work trips

The calculated PA^{a}_{mij} by (Eq. 17) should be converted to O-D trips (origin-destination trips) before analyzing traffic assignment. To do this, we can use the following equation which is described in Easa (1993).

$$OD^{a}_{mij} = \varphi^{a} PA^{a}_{mij} + (1-\varphi^{a}) PA^{a}_{mji} \qquad \text{(Eq. 18)}$$

where OD^{a}_{mij} = number of trips from zone i to zone j by using mode m for purpose a

φ^{a} = proportion of trips originating from a production zone and destined to a attraction zone for trip purpose a

The above O-D trips by purposes and transportation modes can be aggregated by transportation modes for highway traffic assignment and transit trip assignment as following;

$$OD_{mij} = \sum_{a} OD^{a}_{mij}, \quad \forall \text{ m = auto, transit} \qquad \text{(Eq. 19)}$$

where OD_{mij} = number of trips from zone i to zone j by using mode m (auto or transit)

The O-D trips by transportation mode (for example, auto and transit), which are calculated by (eq. 19), are the input data for auto traffic assignment analysis and transit trip assignment analysis. The Frank-Wolf algorithm, which is one of algorithms to solve deterministic user equilibrium assignment, may be applied for highway network assignment with the auto O-D matrix. The Dial's algorithm, which is a algorithm to solve stochastic assignment, or all-or-nothing assignment may be applied for transit network assignment with the transit O-D matrix. Anyhow, any kind of algorithms to solve assignment problem for auto and transit can be applied depending on analyst's preference based on his analysis purpose without affecting this model structure.

This transportation demand analysis sub-model will generate the minimum path travel time between each zone pairs by each transportation mode. And it will also give link flows and link travel time. These results of transportation systems analysis will be used as the input data for the residential location choice sub-model.

3.3 Equilibrium between transportation and housing market system

In housing market system, a stable condition is reached only when no resident can improve his utility by unilaterally changing his or her residential location with given travel time between home and workplace. In transportation system, any traveler can not improve his or her satisfaction of traveling by unilaterally changing travel destinations, transportation modes and routes at equilibrium with given activity allocation. Therefore, the equilibrium between transportation and housing system can be defined as the state of which the given travel times between pair of zones for housing market sub-models are same as the travel times which are the results of transportation sub-model analysis. At the same time, this state also has to be satisfied that the given allocation of activities in transportation sub-models is same as the results of the housing market sub-model analysis. In other words, the input travel time of the residential location choice model is same as the expected travel times which are the outputs of auto traffic assignment and transit assignment analysis in transportation demand analysis sub-model at equilibrium of two systems. The equilibrium housing rents and equilibrium travel time of each link can be calculated by replacing rents and travel times as the newly calculated ones at each iteration, of which method was called an iterative method in this study.

By solving the equilibrium, we can analyze the ripple effects of transportation changes on activity and transportation system and the ripple effects of activity shift on transportation system and activity system. Therefore, this model can analyze a transportation or housing policies while the interactive effects of transportation and housing system can be explicitly reflected into the model. The modified LUTEM model will endogenously generate following variables at equilibrium state : equilibrium housing rents, average travel time between zones, composition of households at a zone by income level, number of trips between zones by trip purposes, market share of each transportation mode, link travel time and speed, level of service at each link of highway and transit line and so on.

4. SOLUTION ALGORITHM : AN ITERATIVE METHOD

Instead of solving simultaneously all of equations in the modified LUTEM model, we can block them into two recursive stages : the first stage solves for all of housing rents, the second stage solves for travel times at user travel equilibrium. Then, an iterative method can be applied to equilibrate transportation system and housing market by using the two stages, recursively. In the first stage, the simultaneous non-linear equations of the housing market equilibrium condition (eq. 7) can be solved using MINPACK program that uses Powell's hybrid method that is a modification of the Newton-Raphson method. In the second stage, well-known algorithms such as Frank-Wolf algorithm and Dial's algorithm can solve travel pattern at user equilibrium. The solving algorithm for the modified LUTEM model can be described as followings;

(step 1) Initialize rents (r_i^0) and average travel times (T_{ij}^0) between zones.
Let iteration number n = 1 and $r_i^n = r_i^0 \quad \forall$ i, and $T_{ij}^n = T_{ij}^0 \quad \forall$ i, j

(step 2) Allocate employees to housing market by using (eq. 1) and (eq. 3)
$P_{h|i}^n P_{i|j}^n = f(r_i^n, T_{ij}^n)$

(step 3) Solve the nonlinear simultaneous equations (eq. 7) to get the housing market equilibrium rents by using (eq. 1), (eq. 3) and MINPACK program.
Let the equilibrium rent r_i^*

(step 4) Calculate the P-A trips for each trip purpose by using (eq. 8) and (eq. 13)
$PA_{ij}^{na} = f(r_i^*, T_{ij}^n) \quad \forall$ a, i, j

(step 5) Calculate the P-A trips by each transportation mode for each trip purpose by using (eq. 15) and (eq. 17)

(step 6) Convert the P-A trips to the O-D trips and sum the O-D trips by transportation mode by using (eq. 18) and (eq. 19)

(step 7) Calculate link volumes and travel time at user equilibrium for auto trips and transit trips by using Frank-Wolfe algorithm and Dial's algorithm, respectively.
Let the average equilibrium travel time between zones T_{ij}^{n+1}

(step 8) Calculate $\varepsilon_{ij} = \dfrac{|T_{ij}^{n+1} - T_{ij}^n|}{T_{ij}^n} \quad \forall$ i, j

Find out $\varepsilon^* = \max(\varepsilon_{ij} : \varepsilon_{ij} = \dfrac{|T_{ij}^{n+1} - T_{ij}^n|}{T_{ij}^n}, \forall i, j)$

(step 9) Compare the error ε^* with tolerance level Ω
If $\varepsilon^* \geq \Omega$, then go to (step 2)
If $\varepsilon^* \prec \Omega$, then stop and get all equilibrium solutions

5. SUMMARY AND CONCLUSION

This study developed a model which can analyze housing polices and/or transportation policies. The model can explicitly consider the ripple interactive effects between housing market and transportation system. The model is a modified LUTEM (Land Use and Transportation Equilibrium Model) which becomes more practical for policy analysis and only focuses on housing market among several land use activities with transportation system. This model has three sub-models ; residential location choice sub-model, housing market equilibrium sub-model and transportation demand analysis sub-model. In the residential location sub-model, a nested logit model was applied. In housing market equilibrium sub-model, the equilibrium housing rents can be calculated at stochastic Walrasian equilibrium condition. In transportation demand analysis sub-model, the well-known traditional four step transportation model was applied except for trip distribution of work-trips.

This study also developed a solution algorithm to solve the modified LUTEM model. An iterative method was applied to solve this equilibrium problem between two systems : transportation and housing market system. This algorithm consists of two stage : one for solving housing market equilibrium condition with given travel times between zones and another for finding out travel pattern at user equilibrium. By using the model developed in this study, the equilibrium housing rents, average travel time between zones, composition of households at a zone by income level, number of trips between zones by trip purposes, market share of each transportation mode, link travel time and speed, level of service at each link of highway and transit line can be analyzed when there are some changes in transportation and/or housing market system by some specific policies.

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