APPLICATION OF LP AND FLP IN A CALL TAXI DISPATCHING SYSTEM

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abstract: In Seoul, elderly and handicapped people or women carrying babies cannot get taxi services easily, since most taxies including "Mobum" which offer high quality services are not operated with a call dispatching system. This paper shows a dispatching system applicable to the Mobum taxi with the application of automatic vehicle location system(AVL). The dispatching system adopts heuristic vehicle assignment algorithms. A linear programming(LP) and fuzzy linear programming(FLP) are applied to the algorithms. The models produce assignments of empty taxies to the passengers who request services during the preset dispatching time period.

1. INTRODUCTION

About 4,900 high quality taxies called "Mobum taxi" are operating in Seoul. Most citizens are positively thinking of the taxi service since the service quality is much better than that of the standard taxi service called "Joong-hyung(means mid-size)". However, only 10% of the Mobum taxies are operated by a call dispatching system. Most of the taxies just wait passengers in front of hotels and airports. Elderly and handicapped people or women carrying babies cannot get the taxi services easily, since they have difficulties to reach the taxies. They are complaining about the Mobum taxi service.

The city of Seoul has been trying to install a dispatching system for all Mobum taxies. Financial allotments between the city and the taxi drivers who own the taxies have not been compromised yet.

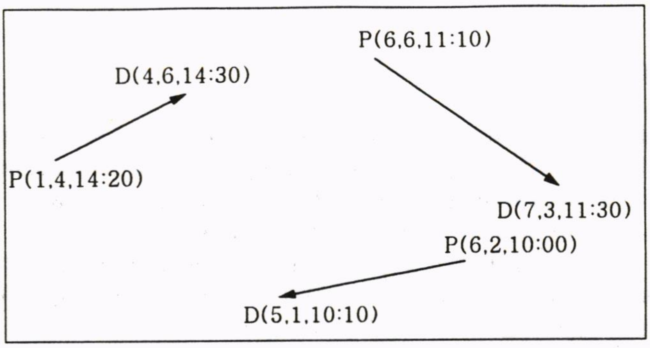
This paper shows a dispatching system applicable to the Mobum taxi or call taxies with the application of automatic vehicle location system(AVL). The dispatching system adopts heuristic vehicle routing(assignment) algorithms. A typical linear programming(LP) and fuzzy linear programming(FLP) are applied to the algorithms.

Firstly, travel characteristics of the demand responsive transportation modes including call taxi are reviewed. Secondly, a dispatching system is explained with vehicle assignment algorithms with application of LP and FLP. Thirdly, numerical examples of the proposed system are shown.

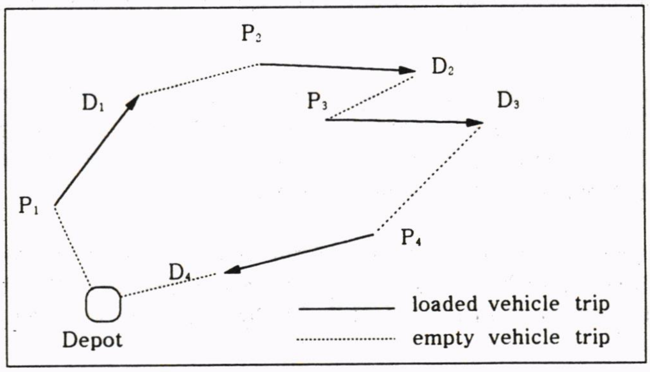
2. CHARACTERISTICS OF THE DEMAND RESPONSIVE TRANSPORT MODE

Generally, the demand responsive transport mode(DRTM) such as dial-a-ride and call taxi operates a dispatching system. The dial-a-ride has a dispatching system operated based on either advanced(typically, one day before) requests or immediate requests by passengers, while the call taxi is dispatched mostly by immediate requests. The request information of the dial-a-ride includes origin, destination, pick-up and drop-off times. The request of the call taxi service consists of origin and destination information only.

A typical passengers' travel pattern of the DRTM is shown in Figure 1. P(i,j,t) means that a passenger located at the point (i,j) is picked up at time t. D(i,j,t) means the passenger is dropped off at the destination, point (i,j) at time t. Also, a vehicle's travel pattern of the DRTM is presented in Figure 2. The pattern is divided into empty vehicle trips which mean no passenger in the vehicle, and loaded vehicle trips which have passengers in the vehicle. In the dial-a-ride operation , passengers who have the same origin or destination may share the same vehicle, while in the call taxi service they may not. This paper focuses on the call taxi operation.



<Figure 1> Passenger Travel Pattern of DRTM



<Figure 2> Vehicle Movement of DRTM

3. COMPONENTS OF THE PROPOSED DISPATCHING SYSTEM

The proposed dispatching system of a call taxi operation consists of three parts, the operating environment including an operating control center, database of passenger and vehicle related information and the dispatching algorithm.

3.1 Operating Environment

Passengers who wish to have call taxi services phone the operating control center (OCC), and inform their origins(pick-up points) and destinations(drop-off points). Also, the locations and service availability(on service or not on service) of taxies are tracked by an automatic vehicle location system(AVL) or informed by taxi drivers, and recorded in the database of the main computer in OCC. Then, the dispatching algorithm explained in the section 3.3 assigns empty taxies to passengers based on the passengers' request information recorded in the computer every 5 or 10 minute depending on the operating policy discussed more in the section 3.2.

This dispatching system is quite different from taxi operation in Korea. Standard taxies(Joong-hyung) are not operated by a call dispatching system. Passengers take taxies by raising their hands when they find empty taxies on the streets. Recently, about 10% of Mobum(high quality) taxies are operated by a call dispatching system that assigns an empty taxi located at the nearest place to the passenger by a dispatcher who monitors the locations of empty taxies on the CRT in OCC.

3.2 Database

Zoning

The target service area is divided into zones. If the number of zones is increasing in the service area, the size of each zone is decreasing. Pick-up and drop-off points of passengers and locations of taxies are coded with corresponding zone numbers.

Dispatching Time Period

The length of the dispatching time period has to be decided. The dispatching time period means time between the times when empty taxies are assigned to passengers. One day is divided into dispatching time periods. If the length of one dispatching time period is assumed to be 5 minutes, we have 288 time periods per day. That is, if the first time period is assumed to be the time between 12:00AM 0 second and 12:04AM 59 seconds, the second time period becomes between 12:05AM 0 second to 12:09AM 59 seconds, and so on. Passengers who request services during the first time period are assigned to empty taxies at 12:05AM. Passengers who request services during the second time period are assigned at 12:10AM. There may exist passengers who requested 12:10 AM services few days ago.

Passenger Information

The operator(or computer) in OCC asks name, pick-up and drop-off points, telephone number of a passenger who calls for a service. Among the information, the pick-up point (address) is converted to corresponding zone number. The drop-off point is not used for the dispatching algorithm. However, the information helps the operator monitor passenger's travel pattern, adjust the number and size of zones properly, and figure out the optimum dispatching time period and number of taxies in the service area. A passenger's name and address are informed to the assigned taxi driver.

Taxi Information

Each taxi's location and service situation (loaded or empty) are monitored by AVL. The location is converted to corresponding zone number. The number becomes input of the dispatching algorithm explained in the next chapter. Also, each taxi's plate number and driver name are recorded. This information is useful for figuring out monthly operating statistics such as total empty and loaded vehicle hours and miles.

Travel Time Between Zones

Travel times between zones in each dispatching time period are estimated by vehicle speed detectors on the highway in the service area. If travel time estimation of every dispatching time period is impossible, average interzonal travel time during the morning or afternoon hours may be used for the dispatching algorithm, but must be updated periodically. If the zone size gets bigger, the estimation error becomes bigger.

3.3 Dispatching Algorithm

Total vehicle hours per day in call taxi operation can be represented by

$$TTHD = \sum_{k}\sum_{t} (LVH_{kt} + EVH_{kt})$$

where $TTHD$: total vehicle hours per day
LVH_{kt} : loaded vehicle hours of taxi k at time period t
EVH_{kt} : empty vehicle hours of taxi k at time period t

Here, the loaded vehicle hour means service time from picking up a passenger to dropping him or her off. Empty vehicle(or empty taxi) hour includes no service time(idle time) as well as time for reaching assigned passengers. The operating revenue increases as the loaded vehicle hours do, and the operating cost can be saved as the empty vehicle hours decrease. The taxi operator wishes to decrease total empty vehicle hours as low as possible, and to increase total loaded vehicle hours. In Korea, owners of the Mobum taxies are drivers themselves. They want to reduce total empty vehicle hours by introducing a control center with a dispatching system.

The following proposed dispatching algorithm minimizes total empty vehicle hours under the given passenger demand and empty taxi supply in a given time period. If the passenger demand and vehicle speeds are fixed, the loaded vehicle hours are constant. Here, we have to consider the length of time period, t. The length of time period is related to waiting time of a passenger who requests a service. If the dispatching time period is assumed to be 5 minutes, the maximum waiting time includes the dispatching time period, 5 minutes, the computing time of vehicle assignment by the algorithm discussed below and travel time from the location of assigned empty taxi to the pick-up point of the assigned passenger. If the computing time of vehicle assignment, which depends on the number of zones in the service area, is assumed 1 minute and the maximum travel time between zones in the service area is 10 minutes, the maximum waiting time becomes 16 minutes. Therefore, the maximum waiting time is mainly decided by the size of service area and the dispatching time period.

Application of LP

Minimizing total empty vehicle(taxi) hours at a time period can be considered as a transportation problem of linear programming. The locations of empty taxi become the supply side of the problem, while the locations of passengers who request services are the demand side.

Then, given the number of empty taxies and passengers by zone at time period t, empty taxies can be assigned to passengers by minimizing total empty taxi hours. When total number of empty taxies is greater than total passengers at time period t, the following model can be adopted.

$$\text{Min.} \quad TEVH_t = \sum_i \sum_j \{VT_{ijt} \times X_{ijt}\}$$

$$\text{S.T.} \quad \sum_j^{zn} X_{ijt} = EV_{it} \qquad i = 1,2, \text{-----} zn$$

$$\sum_i^{zn} X_{ijt} = P_{jt} \qquad j = 1,2, \text{-----} zn$$

$$\text{Here,} \quad \sum_i^{zn} EV_{it} \geq \sum_j^{zn} P_{jt}$$

$TEVH_t$: total empty taxi hours at time period t
VT_{ijt} : taxi travel time between zone i and j at time period t
X_{ijt} : number of empty taxies assigned from zone i to zone j at time period t
EV_{it} : number of empty taxies in zone i at time period t
P_{jt} : number of passengers in zone j at time period t
zn : number of zones

Sometimes, $\Sigma EV_{it} < \Sigma P_{jt}$ may happen. In this case passengers who are in $\Sigma EV_{it} = \Sigma P_{jt}$ based on first-request first-serve are assigned first by the above LP model. Leftover passengers, $LP_{it} = \Sigma P_{jt} - \Sigma EV_{it}$, can be served at the time period, t+1. If the waiting time of the leftover passengers is serious, empty taxies can be preferentially assigned to them by the nearest neighbor algorithm. The calculation time of the LP model mainly depends on the number of zones.

Application of FLP

In the above LP model, travel time between zones, the number of empty taxies in each time period, and the number of service requested passengers are assumed to be fixed or precisely known numbers. The number of passengers and empty taxies at time period t may be assumed to be known exactly, since cancellation or no-show cases are rare in the field and the number of empty taxies can be monitored accurately just before assignment. However, travel time between zones varies every minute in Seoul. The time cannot be constant but uncertain or fuzzy. In this section the previous LP model is converted to a fuzzy linear programming (FLP) model as the travel time between zones is assumed to be a fuzzy number.

In 1960's the fuzzy set theory is introduced and the theory has been applied to uncertainty in operations research. The theory expands the existing theory(crisp) dealing with randomness into considering vagueness, that is, fuzziness in real life problems. Many people such as Zimmermann, Tanaka and Asai have studied the application of fuzzy theory in mathematical programming. Zimmerman (1978) applied the fuzzy theory to availability resources (right hand sides of constraints) in an LP model. Tanaka and Asai(1984) introduced fuzziness in coefficients of the objective function in LP model. Different types of FLP problems have been analyzed depending on the nature of fuzziness in the problem. For the problem with fuzzy numbers in the right-hand side of the constraints, Zimmerman transformed the FLP problem into an ordinary LP problem assuming a linear membership function for each fuzzy number. The LP model maximizes the level of satisfaction that is defined for each objective and constraint in a linear form. The problem with fuzzy numbers for the parameters of the constraints and the objective has been analyzed by Tanaka and Asai(1984), and Delgado, Verdegay and Vila(1989). Tanaka and Asai used a two-step approach that finds the maximum satisfaction of the fuzzy inequalities of the constraints first and then solves the unknowns.

In this paper, travel time between zones is assumed to be fuzzy number, and the parameters of the objective in the LP model are assumed to be fuzzy numbers. Therefore, Tanaka and Asai(1984)'s formulation and solution approach with fuzzy numbers for the parameters of the objective were adopted. If fuzziness in the parameters of the objective in LP model (1), the model can be formulated as follows:

$$\text{Min} \quad \sum_{i}\sum_{j} \{\underset{\sim}{VT}_{ijt} \times X_{ijt}\} \le \underset{\sim}{G}_t \text{ -------------------------------- (1)}$$

$$\text{S.T.} \quad \sum_{j}^{zn} X_{ijt} = EV_{it} \qquad i = 1,2, \text{ - - - - - } zn \text{ ------------------- (2)}$$

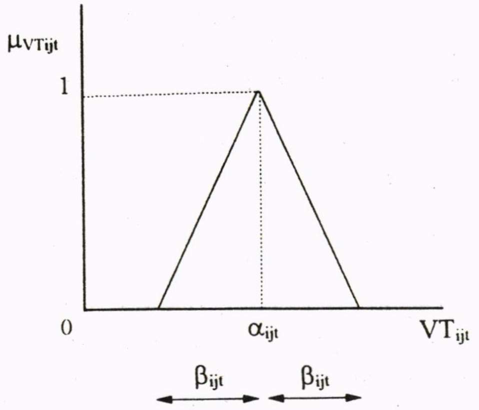
$$\sum_{i}^{zn} X_{ijt} = P_{jt} \qquad j = 1,2, \text{ - - - - - } zn \text{ ------------------- (3)}$$

The fuzzy objective of equation (1) can be expressed as;

$$\underset{\sim}{Y}_t = \underset{\sim}{VT}_{00t} \times X_{00t} - \underset{\sim}{VT}_{11t} \times X_{11t} - \underset{\sim}{VT}_{12t} \times X_{12t} \text{ - - - - - - - } \ge 0 \text{ ------- (4)}$$

where $VT_{00t} = G_t$ and $X_{00t} = 1$.

Here, G_t is the total empty taxi hours decided by the operation policy. We can assume that the membership function of the fuzzy number, $\underset{\sim}{VT}_{ijt}$ is the symmetric triangular type, $\underset{\sim}{VT}_{ijt} = (\alpha_{ijt}, \beta_{ijt})$ shown in the following figure .



<Figure 3> membership function of $\tilde{VT}_{ijt}$

α_{ijt} is the center value of the triangle and β_{ijt} is the spread around the center. Then, the value of Y_t in equation (4) becomes a triangular fuzzy number and it can be expressed using the standard arithmetic operation of triangular fuzzy numbers like equation (5).

$$Y_t = (\alpha_{00t} - \sum_i\sum_j \{\alpha_{ijt} \times X_{ijt}\},\quad \beta_{00t} + \sum_i\sum_j \{\beta_{ijt} \times X_{ijt}\}) \quad \text{------------ (5)}$$

$$= (\alpha_{Yt}, \beta_{Yt})$$

A measure that Y_t is greater than 0 at a degree h is satisfied if $\mu_{Yt}(0) \leq 1\text{-}h$ and $\alpha_{Yt} \geq 0$, where μ_{Yt} is the membership function of Y_t. Then, it can be shown that $\mu_{Yt}(0) = 1-(\alpha_{Yt}/\beta_{Yt})$ and $(\alpha_{Yt} - h\times\beta_{Yt}) \geq 0$. (see Tanaka and Asai, 1984, Shinya Kikuchi, *et al*, 1991) For the policy decision maker to be satisfied with the solution at a given level, h (level of satisfaction), it is required that the solution satisfies with the objective and the constraints by at least the given value of h. Then, the problem can be reformulated as follows:

Max h

$$\text{S.T} \quad (\alpha_{00t} - \sum_i\sum_j \{\alpha_{ijt} \times X_{ijt}\}) - h \times (\beta_{00t} + \sum_i\sum_j \{\beta_{ijt} \times X_{ijt}\}) \geq 0 \quad \text{---------- (6)}$$

$$\sum_j^{zn} X_{ijt} = EV_{it} \qquad i = 1,2, \text{- - - - -} zn \quad \text{------------------------------ (7)}$$

$$\sum_i^{zn} X_{ijt} = P_{jt} \qquad j = 1,2, \text{- - - - -} zn \quad \text{------------------------------ (8)}$$

Equation (6) can also be expressed as;

$$\sum_i\sum_j (\alpha_{ijt} + h \times \beta_{ijt}) \times X_{ijt} \leq \alpha_{00t} - h \times \beta_{00t} \quad \text{---------- (9)}$$

Since the above problem is non-linear optimization due to the form of multiplication of h and X_{ijt} in equation (9). Tanaka and Asai(1984)suggested the following technique:

1. Give an initial value of h and determine if the feasible solution of X_{ijt} exists for (7), (8) and (9). This can be found by the operation of the first-step on the two-step simplex method.
2. Increase the value of h until the feasible solution space ceases to exist, let h* be the maximum value of h.
3. For h* determine the optimum X_{ijt}, by solving LP problem in which minimizing the left-hand side of equation (6) is the objective and equations (7) and (8) are constraints.

Here, the optimum values of X_{ijt}'s are changed by how to assume the fuzzy numbers, $VT_{ijt}=(\alpha_{ijt}, \beta_{ijt})$, that is the parameters of the objective, and h, the degree of satisfaction. α_{00t} means total empty taxi miles set by the operator or the decision maker, and β_{00t} means upper and lower bound of α_{00t}. h means the degree of satisfaction of the operator. The value of h can be differently assumed for different membership function of VT_{ijt}. Fuzziness in both supply, empty taxies and demand, passengers can be easily expressed by membership functions which Zimmermann(1976) proposed, where EV_{it} and Pit are the fuzzy set representing the decision maker's level of acceptability for the supply lower than EV_{it} and the demand greater than P_{it}. However, if EV_{it} and P_{it} are assumed to be fuzzy numbers, the solutions of FLP, that is, Xijt's are not integer. For the integer solutions, the branch and bound algorithm may be applied, whose computing time is much longer than that of the simplex method.

4. EXAMPLES

Zoning an Imaginary Service Area and Setting up Dispatching Time Period

An imaginary service area is divided into 6 zones. If it is assumed that taxies operate 24 hours a day and the dispatching time period is 5 minutes, one day is divided into 288 dispatching time periods (24hours × 12 time periods /1 hour = 288 time periods). If the dispatching time period from 12:00 AM to 12:05 AM is the first dispatching time period, the time period between 6:00AM and 6:05AM becomes the 73rd dispatching time period. That is, passengers who request services during this time period are assigned to empty taxies at 6:05AM by the dispatching algorithm.

Maximum Passenger Waiting Time

In the taxi operation one of the most important service policy is the passenger's waiting time. The operator should inform the maximum waiting time to the passengers and let them estimate the arrival times of assigned taxies. Generally, a passenger wishes to be served right after he or she requests a service. In this system, he must wait for the time until a taxi is assigned to him(the maximum time is equal to the preset dispatching time period, which is 5 minutes in this example, however, the calculation time of the dispatching algorithm is ignored.) and for the taxi's travel time from its location where it is assigned to his pick-up point. If the maximum travel time in the service area is assumed to be 10 minutes, the maximum waiting time of a passenger who requests a service at 6:00 PM is going to be 15 minutes since an empty taxi is dispatched(assigned) at 6:05AM. Also, a passenger who requests a service at 6:06 AM is going to take a taxi within 6:20 AM because a taxi is assigned at 6:10 AM and the maximum travel time in the service area is 10 minutes. Therefore, the maximum passenger waiting time depends on the size of the service area and the length of the dispatching time period decided by the operating policy.

Passenger and Vehicle Information

We need database of empty taxies and passengers who request in each dispatching time period. If the time period is assumed to be the 73rd dispatching time period between 6:00 AM and 6:05 AM, the necessary data will be the number of empty taxies by zone in the service area at 6:04 AM 59 seconds and the number of passengers by zone from 6:00 AM 1 second to 6:04 AM 59 seconds. For example, during the time period the number of passengers and empty taxies at 6:04 AM 59 seconds are assumed to be shown as the following figure.

Zone No.	1	2	3	4	5	6
No. of Taxies	2	5	4	2	4	5
No. of Passengers	4	2	3	3	2	4

<Figure 4 > locations and numbers of empty taxies and passengers at the 73rd time period

For example, in zone 2, 5 empty taxies are available, and 2 services are requested.

Interzonal Travel Time

Interzonal travel time database are also necessary to apply the dispatching algorithms. By using speed detectors along the road, interzonal travel time between zone centriods during each time period may be estimated on the real-time basis. In this example, interzonal travel time during the 73rd dispatching time period is assumed to be as follows.

	1	2	3	4	5	6
1	5	10	15	5	10	20
2	10	5	10	15	10	15
3	15	10	5	20	15	10
4	5	15	20	5	10	15
5	10	10	15	10	5	10
6	20	15	10	15	10	5

<Figure 5> Interzonal travel time

Intrazonal travel time is also considered. The intrazonal travel time is assumed to be 5 minutes for all zones in this example. Practically, the intrazonal travel time depends on the size of each zone. The maximum passenger waiting time is decided by the preset dispatching time period, 5 minutes, intrazonal travel time, 5 minutes and travel time between the location of assigned empty taxi and the passenger.

Application of LP Model

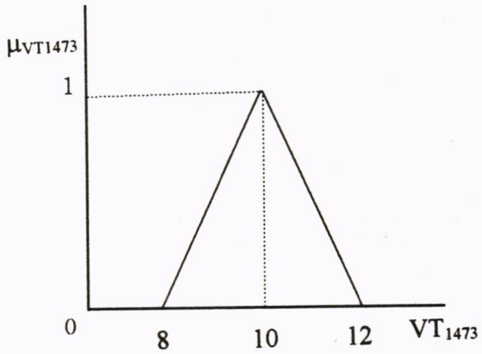
In this example, the number of available empty taxies is assumed to be greater or equal to the number of requested services during the time period from 6:00 AM to 6:05 AM. If the LP model is applied, the total empty taxi hours are estimated to be 110 minutes and the dispatching strategy follows the table below.

	1	2	3	4	5	6
1	2	1			1	
2		2				
3			3			
4				2	1	
5					2	
6			1			3

<Figure 6> Dispatching Result by LP model

<u>Application of FLP model</u>

Interzonal travel time, VT_{ijt} is assumed to be fuzzy numbers, which have the triangular membership function, $(\alpha_{ijt}, \beta_{ijt})$, where t is 73, which means the 73rd dispatching time period in this example. β_{ijt} is assumed 20% of α_{ijt} for all i and j. This means the operator permits the maximum error or uncertainty(20%) of estimated interzonal travel time, α_{ijt} during the time period. The value of β_{ijt} can be assumed differently depending on zone i and j, and the dispatching time period. In this example, the membership function of VT_{1473} from zone 1 to zone 4 during the 73rd time period becomes $VT_{1473} = (\alpha_{1473}, \beta_{1473})$, where $\alpha_{1473} = 10$, $\beta_{1473} = 2$ as the following figure.



<Figure 7> membership function of VT_{1473}

$VT_{0073} = (\alpha_{0073}, \beta_{0073})$ stands for total empty taxi hours during the 73rd time period. In this example, it is assumed that α_{0073} is 100 minutes, and β_{0073} is 20% of α_{0073}. That is, the operating policy allows total empty taxi hours less than 120 minutes. If the FLP model is applied, the total empty taxi hours are 114.4 minutes during the 73rd time period, and the empty taxies are assigned to passengers as the following table with the degree of satisfaction, h, 80%(0.8) which is arbitrarily set .

	1	2	3	4	5	6
1		1		2	1	
2		2				
3			3			
4	2				1	
5					2	
6			1			3

<Figure 8> Dispatching Result by FLP model

This result shows that uncertainty of the interzonal travel time is considered as fuzzy number, and the FLP model produces relaxed result from that of the LP model with the 80% degree of satisfaction of the operator.

5. CONCLUSIONS

A dispatching system of the call taxi was proposed in this paper. The dispatching models in the system produce assignments of empty taxies to passengers who request services during the preset dispatching time period. The models adopted linear programming and fuzzy linear programming theory. The models can be easily solved by PC level computers. Application of fuzzy theory helps the dispatcher cope with uncertainty of operating parameters such as interzonal travel times. The application of fuzzy theory can be easily expanded to dealing with uncertainty of the number of empty taxies and passengers at each time period, which may not be serious in the real field. The proposed dispatching system may be tested in a real taxi operation and compared with the existing dispatching methodologies such as the nearest neighbor algorithm.

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