A RAMSEY PRICE EQUILIBRIUM MODEL AND ITS COMPUTATIONAL PROCEDURE

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abstract: The Ramsey Price Equilibrium (RPE) model is proposed by Miyagi et al. 1992. This model determines the optimal fares for urban transits under the competition between automobiles and urban transits. The purpose of this paper is to investigate a computational method for the RPE model. We apply a nonlinear sensitivity analysis based on the penalty function method. Then we examine our method through numerical experiments and discuss about the availability of the RPE model.

1. INTRODUCTION

It has been established that using marginal cost pricing, resources are allocated fairly and social welfare are maximized. However if the firm set price equal to marginal cost in the context of economy of scale, the firm would not be sustainable for deficit. This situation is usually observed in transit pricing for the most of middle-size cities of Japan. In this case the second best pricing rules like a Ramsey pricing (1927) are important, because the marginal cost pricing is not useful for above reason. If a firm sets price using the Ramsey pricing rule, then the social welfare subjected to break-even constraints would be maximized.

An application of Ramsey rule to public transit pricing was conducted by Train(1977) for AC transit and BART, however, a network competition between transit and auto arising from the pricing rule is not taken into account in that application. Miyagi et al.(1992) proposed a Ramsey Price Equilibrium Problem (RPEP) in which the Ramsey pricing rule is restructured within the framework of multimodal network equilibrium and the RPEP is formulated as a bilevel programming problem. This model consists of the upper problem, being defined as maximization problem of the social welfare using the Ramsey rule, and the lower problem with multimodal network equilibrium. Then Miyagi and Suzuki (1995) further extended the RPE model to include the variational inequality formulation for the binary modal choice / assignment model in the lower problem and proposed a computational procedure based on the nonlinear sensitivity analysis for the restricted variational inequality proposed by Tobin and Friesz (1988).

Other studies also have been performed to apply bilevel programming problem to transportation policy analysis, where the transportation network equilibrium problem is included in the lower problem. For example, Fisk (1984) shows that Nash equilibrium solution possibly provides a different solution from that generated by Stackelberg equilibrium. Yang (1994) proposed a penalty function method to solve bilevel programming formulation for general freeway-arterial corridor systems.

This paper has three major purposes. First, it aims at identifying the difference between the RPE model and the traditional Ramsey pricing rule through some numerical experiments. The second purpose is to develop a numerical calculation method in line with the nonlinear sensitivity analysis proposed by Miyagi and Suzuki (1996). Finally, we want to check whether the RPE model can be used to search the optimal subsidy level from the view point of social welfare which is invested in construction of new transit systems.

2. RAMSEY PRICE EQUILIBRIUM MODEL

2.1 Notation Concerned with Network Equilibria

The expanded network is represented by a directed graph G(N, L) where N is the set of nodes and L is the set of directed links. The set of all origin / destinations (O/D) pair is designated by I. The network permits the flow of vehicles and transit passengers on links. The transit vehicles follow fixed itineraries. The nodes n, $n \in N$, represent origins, destinations and intersections of links; the links a, $a \in L$, represent the road and transit infrastructure of the urban area. The modes are designated by index m which is 1 for the automobile mode and 2 for the transit mode. The individual user cost c_a^m for travel by mode m on link a are given by monotone, continuous and differentiable functions of the link flow of each mode, v_a^1 or v_a^2 : $c_a^1(v_a)$, $c_a^2(v_a)$

The origin to destination demands, q_i^m , $i \in I$, for each mode m may use directed paths k, $k \in \Lambda_i^m$, where Λ_i^m is the set of paths, ($\Lambda_i^m \neq \phi$), available for mode m and O/D pair i. The total origin to destination demands by both modes , $q^1 \in R^I$ and $q^2 \in R^I$, are given by a constant matrix $\bar{q} \in R^I$, where for OD conservation equation is given as :

$$q_i^1 + q_i^2 = \bar{q}_i \, , i \in I \tag{1}$$

The flows on paths k, h_k, satisfy conservation of flow and nonnegativity.

$$\sum_{k \in \Lambda_i^m} h_k = q_i^m, \quad \forall i \in I, m = 1,2 \ \ and\ h_k \geq 0 \tag{2}$$

The link flows v_a^m are given by

$$v_a^m = \sum_{i \in I} \sum_{k \in \Lambda_i^m} \delta_{ak} h_k \, , m = 1,2 \, , a \in L \tag{3}$$

where

$$\delta_{ak} = \begin{cases} 1 & \text{if link } a \text{ belongs to path } k \\ 0 & \text{otherwise.} \end{cases}$$

The cost of each path $C_k(v)$ is the sum of the user costs of the links in the path

$$C_k(v) = \sum_{a \in A} \delta_{ak} c_a^m(v_a), \quad k \in \Lambda_i^m, \forall i \in I, m = 1,2. \tag{4}$$

Let $\mu_i^m(v)$ be the least cost using mode m for a O/D pairs i:

$$\mu_i^m(v) = \min._{k \in \Lambda_i^m} C_k(v), \quad \forall i \in I, m = 1,2. \tag{5}$$

We now consider the case where a mode choice function $G_i(w_i)$ is the logit model, that depends on the travel costs by the two modes via their difference w_i, $w_i = \mu_i^1(v) - \mu_i^2(v)$. For each centroid pair, one can eliminate the transit demand using(2) and it is possible to derive function G so that the auto demand can be obtained from $q_i^1 = q_i G_i(w_i)$. It is assumed that $G_i(w_i)$ is a strictly decreasing function with inverse $W_i(q_i^1 / \overline{q}_i)$. Since $\overline{q}_i$ is constant for a given i we refer to the inverse function as $W_i(q_i^1)$.

The binary mode choice/assignment model is formulated by supposing that no traveler has the incentive to change mode

$$\mu_i^{1*} - \mu_i^{2*} = W_i(q_i^{1*}), \quad i \in I \tag{6}$$

and that for each mode the path choice satisfies Wardrop's user optimized behavioral principle.

$$C_k^* - \mu_i^{m*} \begin{cases} = 0 & \text{if } h_k^* > 0, \\ \geq 0 & \text{if } h_k^* = 0, \end{cases} \quad k \in \Lambda_i^m, \forall i \in I, m = 1,2 \tag{7}$$

subject to the feasibility constraints (1),(2) and (3).

2.2 Formulation

We formulate Ramsey price equilibrium problem (RPEP) with the application of the binary choice / assignment model described in the previous section. Mutual exclusive transit networks are assumed to handle a multimodal network equilibrium problem within the context of the binary modal choice formulation. While demand shares between private automobile and public transit systems are determined by a logit modal split function, demand for each public transit available is determined as the path flow on each public transit route with user equilibrium mechanism. The break-even constraints are imposed on transit routes as a whole.

Algebraically, RPEP can be written as follows:

[RPEP]

[U0]

$$Max.\ \Pi_i(h,p) = \theta \sum_{i \in I} \bar{q}_i \ln \sum_{m=1,2} \exp\left[\frac{(a_i - \mu_i^m)}{\theta}\right] + \sum_{i \in I} \sum_{k \in \Lambda_i^m} \left[p_k h_k - T_k(h(p))\right] \tag{8a}$$

$$\text{s.t.} \ \sum_{i \in I} \sum_{k \in \Lambda_i^m} \left[p_k h_k - T_k(h(p))\right] = K \tag{8b}$$

[L0]

(h,q) is the solution for the following binary mode choice / assignment model proposed by Florian and Spiess (1983):

$$Min.\ Z = \sum_{a \in \Lambda} \int_0^{y_a^1} c_a^1(x)dx + \sum_{a \in \Lambda} \int_0^{y_a^2} c_a^2(x)dx - \sum_{i \in I} \int_0^{q_i^1} W_i(y)dy \tag{9}$$

subject to (1),(2) and (3).

In the objective function of [U1], the first and second terms represent consumer surplus and producer surplus, respectively. The constraint function describes the brake-even condition for public transit service provider, in which p_i denotes a price for the service i, K the money transfer from the government to each public transit service and $T_k(h(p))$ is the joint cost function of producing public transit services with respect to a kth transit line.

For [L0], the necessary conditions for the lower problem can be replaced by the following equations:

$$q_i^1 = \frac{\bar{q}_i}{1 + \exp\left[(\mu_i^1 - \mu_i^2 + a_i)/\theta\right]} \tag{10a}$$

$$(C_k - \mu_i^m)h_k = 0 \quad k \in \Lambda_i^m, i \in I, m = 1,2 \tag{10b}$$

$$C_k - \mu_i^m \geq 0 \qquad k \in \Lambda_i^m, i \in I, m = 1,2 \tag{10c}$$

$$\sum_{k \in \Lambda_i^1} h_k = q_i^1 \tag{10d}$$

$$\sum_{k \in \Lambda_i^2} h_k = \bar{q}_i - q_i^1 \tag{10e}$$

$$h \geq 0 \tag{10f}$$

3. MODELS

We consider a network which consists of three centroids and one intermediate node connected by seven links. The centroids 1 and 2 are suburb areas and 4 is a central business district. It is assumed that 20 thousand people is traveling from both suburb areas to the central district. LRT is planed by public transportation agency in the area. If LRT will be completed, OD pair 1-4 will be connected by public transits (bus and LRT) and automobile networks of which each mode consists of only single route. OD pair 2-4 is connected by two automobile routes, in which one is a short and narrow route and another is a long and wide one. The bus and automobile flow mutually independent on links (1,3,6,7). The interaction of automobile trips for the two OD pairs (1-4 and 2-4) occurs on link 3 where two routes, 1 and 2, overlap.

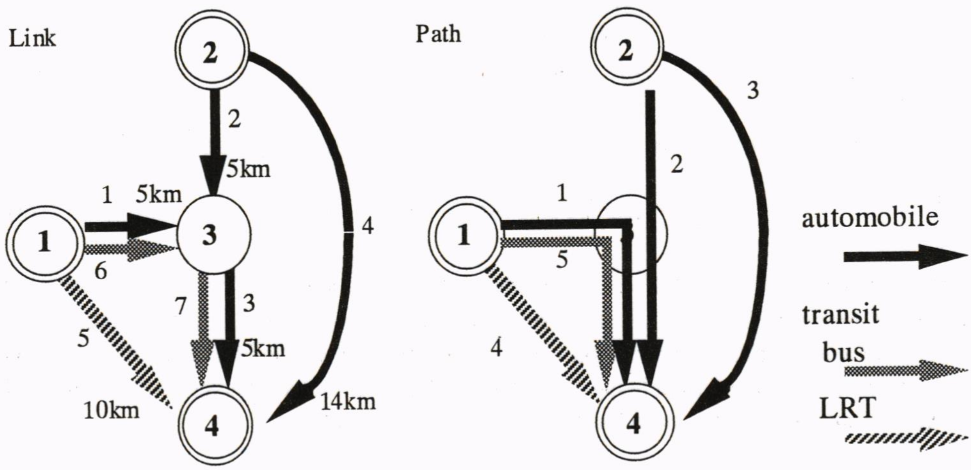

Figure 1 Simplified Urban Transportation Network

Algebraically, RPEP for the simplified urban transportation system depicted in Fig.1 can be written as follows:

[RPEP-1]
[U1]

$$Max.\,\Pi(h,p) = A + \theta q_{14} \ln \sum_{m\in 1,2} \exp\left(-\frac{\mu_{14}^{m}}{\theta}\right) - q_{24}\mu_{24}^{1} + \sum_{k\in 4,5}\left[p_k h_k - T_k(h(p))\right] \quad (11a)$$

$$s.t. \sum_{k\in 4,5}\left[p_k h_k - T_k(h(p))\right] \le K \quad (11b)$$

[L1]

the same as [L0]

where $A = \sum_{i\in I} \bar{q}_i a_i$ represents the total transportation benefit for the total trip.

The objective function of U1 represents a maximization problem of social welfare. The first three terms show the consumer surplus for travel between all OD pairs, the last term represents producer surplus. The constraint is a break-even condition which denotes the company to make zero profit given a constant level of subsidy K.

We have to specify the functional forms of $T_k(\cdot)$ and link cost functions which are used in the above RPE model. The relationship between travel times and flows on road links are described by BPR type function:

$$t_a(v_a) = t_{a0}\left\{1 + 0.15(v_a / Q_a)^{4.0}\right\},\ a \in 1,2,3,4 \tag{12}$$

$t_a(v_a)$:travel time on link a
t_{a0}:free flow travel time on link a
v_a:flow on link a
Q_a:capacity of link a

and a link travel time function for bus is described by a monotone increase function:

$$t_a(v_a) = t_{a0}\left\{1 + 0.15(v_a / 40)\right\} \quad ,\ a \in 6,7 \tag{13}$$

The travel time of LRT will be fixed regardless of LRT flow level. The travelers using bus and LRT between OD pair 1-4 are assumed to be in the user equilibrium situation with respect to generalized transportation costs, the composite cost of transit fares and travel time, with the value of time being 3.3($/min.). The operational characteristics concerned with LRT and bus are shown in Table 3.1.

Table 1 Data of Numerical Example

	LRT	BUS
Route Distance	10km	10km
Frequency	20km/hr	10km/hr
Operation Times	7:00 - 22:00	7:00 - 22:00
Total Service Distance	1200km/day	300km/day
Construauction Costs	1.67×10^7US$/km	——
Subsidies	65% of Cons.Cost	$8.33 \times$ passenger$^{0.7}$us$

(※exchange rate 1 us$ = 120 yen)

The cost of producing services for both of LRT and bus may be described by the following Cobb-Dauglus function:

$$T(h_4, h_5) = \left(FC_B + FC_L\right) + VC = \left(FC_B + FC_L\right) + \tau P^{\alpha_1} Q^{\alpha_2} L^{\alpha_3} \tag{14}$$

where FC_B:Fixed cost for bus (FC_B=0 is assumed)
FC_L:Construction cost for LRT (1.67×10^7 us\$)
VC :Variable cost
P : Operating costs per day (500\$/day)
Q : passenger per day
L : total operating distance (1500 km/day)
$\tau, \alpha_1, \alpha_2, \alpha_3$: parameters (0.001 , 0.2 , 2.0 , 0.5)

Parameters are estimated using data of 81 bus companies in Japan. (Miyagi and Nakatsuhara, 1996). We introduce the subsidy rate β defined by the following expression:

$$K = \beta FC_L. \tag{15}$$

4. NUMERICAL EXAMPLE

In the first place step the ramsey pricing problem(RPP) for transit fares is dealt with the single mathematical programming to compare with Ramsey Price Equilibrium model. The difference between RPP and RPEP lies in whether the automobile network congestion is taken into account or not. In the second place we apply a slightly modified procedure of sensitivity analysis proposed by Miyagi and Suzuki (1996) to solve the optimal fares in Stackelberg equilibrium. Furthermore we conduct parametric analysis to fined out the optimal level of subsidy by changing the subsidy rate defined by (15) over a specified range.

4.1 Solving the Ramsey Pricing Equilibrium Problem without Road Congestion

Supposing that link travel times on all links are given and fixed, then [RPEP-1] is reduced to the single nonlinear optimization problem where the objective function is maximized subject to the break-even constraint (12b) and the logit type modal split function (10a).The assumption implies that the transit service provider knows that optimal user's behavior for transportation mode choice is affected by the fare of each transit mode. An argumented quasi-newton's method with penalty function is used to solve the RPP; the optimal transit fares are determined to minimize the objective function (4-4a) with penalty function defined by constraints (4-4b)-(4-4e).

In this problem the variables are transit fares (p_4,p_5) and flows (h_4,h_5). If for given bus fare we put $q_M = q_4 + q_5$ (mass transit demand), then [RPEP-2] can be depicted in three dimension space as is shown in Fig.2, where AC, MC, DD are average cost, marginal cost, transit mode demand surfaces. The cost-trip relationship at 900 bus trips is shown in Fig. 3. This figure tells us that the marginal cost pricing (the point A) makes the provider deficit because the cross point is under the corresponding average cost (the point B). The Ramsey equilibrium problem provides us that the optimal LRT fares and flows, 2.60\$ and 3752 trips, and for bus those are 2.23\$ and 984 trips (the point C).

[RPP]

$$\min.\Pi = -(20000\times\theta\ln\left[\exp\left(\frac{-p_1-\omega\mu_{14}^{1*}}{\theta}\right)+\exp\left(\frac{-p_4-\omega t_4}{\theta}\right)\right]$$

$$+p_4h_4+p_5h_5 \tag{15a}$$

$$-0.0001\times 60000^{0.2}\times 1500^{0.5}(h_4+h_5)^{2.0}-7.6\times 10^5$$

$$+1000h_5^{0.7}-20000\,\omega\mu_{24}^{1*}$$

$$s.t.\quad p_4h_4+p_5h_5-0.0001\times 60000^{0.2}\times 1500^{0.5}(h_4+h_5)^{2.0}-7.6\times 10^5\le 0 \tag{15b}$$

$$h_4+h_5=\frac{20000}{1+\exp\left[\frac{1}{\theta}\left(-p_1-\omega\mu_{14}^{1*}-(-p_4-\omega t_4)\right)\right]} \tag{15c}$$

$$\bar{q}=\sum_{k\in 1,4,5}h_k=20000 \tag{15d}$$

$$h_5=\frac{10\times 400}{\omega t_4\times 0.15(p_4+\omega t_4-p_5-\omega\mu_b^{14*})} \tag{15e}$$

p_k : fare of k th path (k correspond to each mode, $p_1=0$ is assumed)
t_k : travel time of k th path
h_k : flow of k th path
μ_i^{m*} : the given least travel time for mode m between OD pair i
ω : the value of time
θ : parameter (=100)

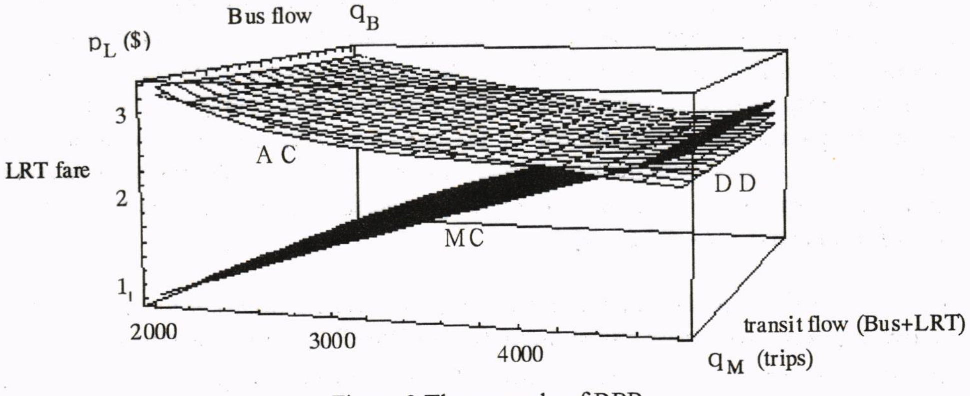

Figure 2 The example of RPP

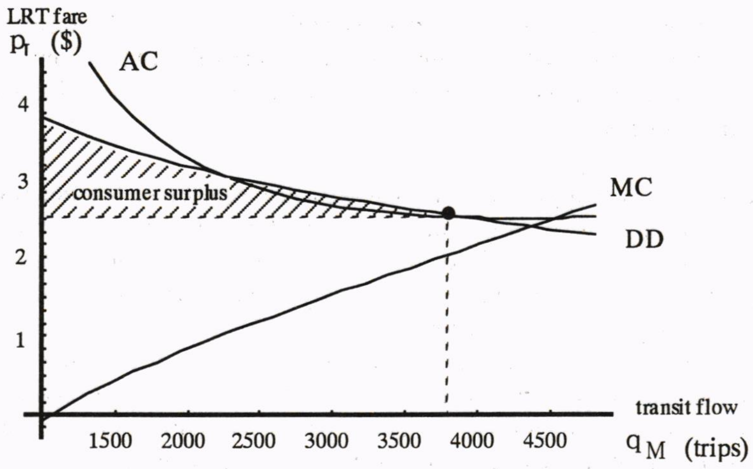

Figure 3 The section of Fig. 2 at 900 bus trips

We can easily see that the optimal fares derived from [RPP] are not valid when road congestion prevails. If we calculate a congested network equilibrium given these fares, the modal split function shifts upwards from the original one. The reason is that when transit fares are raised, some users change transportation mode from transit to automobile so that the congestion level changes on automobile network. This may affect on the transit agency's decision-making on the fare. Therefore, the optimal decision on public transit fares consistent with transit demand should be made not only on user's behavior but also on changes of the congestion level induced from the changes of transit fares.

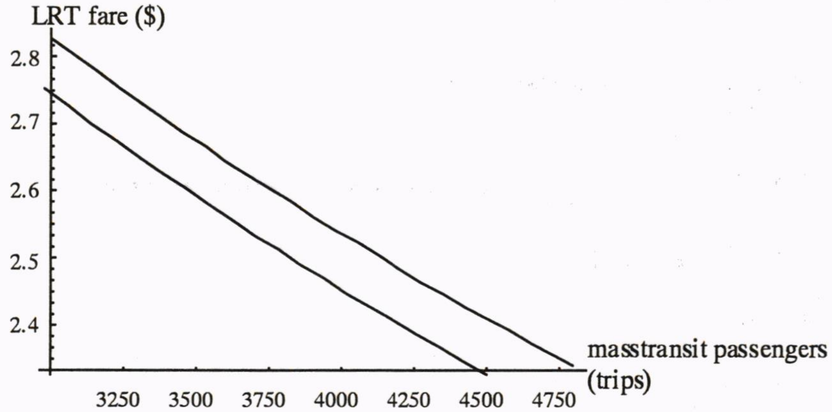

Figure 4 The shift of modal split function due to congestion of automobile network

4.2 Solving the Ramsey Price Equilibrium Problem with road congestion

In the case of solving Stackelberg equilibrium, it is implicitly assumed that the transit service provider seeks an optimal solution with information on how much modal choice and the resultant network flows is changed due to an addition increase or decrease for the current fare level. In order to treat the simultaneous changes in modal choice and road network congestion, network equilibrium conditions (10) must be incorporated into a optimization program [RPP-1]. However, since network equilibrium conditions are generally described by the complementarity condition, it is difficult to handle it as one of the penalty functions.

One needs methods to approximate numerically a new equilibrium solution resulting from a change of decision variables in the upper problem. If it is already known that a solution of nonlinear equation systems (10) exists and is uniquely determined, then a parametric optimal solution q* and Lagrange multiplier may be represented by implicit functions as price vector being independent variable (Simizu 1982). Thus, once the derivatives of the lower level is obtained with respect to the decision variables of the upper level, a penalty function method can be utilized to solve [RPEP-1]. For this purpose, the nonlinear sensitivity analysis is useful because that any parameter perturbation will generally results in the network equilibrium solution and that this type of sensitivity analysis requires the calculation of decision variables and constraint multipliers with respect to perturbation parameters (Fiacco 1983; Tobin and Friesz 1988).

The results are shown as figure 5 to 9. Since the calculation converges speedy to local minimum at 9th iteration, the nonlinear sensitivity analysis method is useful for RPEP in a case of this example. In the equilibrium the fares of LRT and bus are 2.63$ and 2.26$, flows are 2556 and 984 trips. These solutions are calculated by the provider considering not only mass transit but also automobile network taking into consideration , the optimal fares are decided for whole of the urban transportation system. Therefore from the view of the making good use of pricing to control whole of the urban transportation system, to be given the subsidies for the transit provider operating the RPEP is justified.

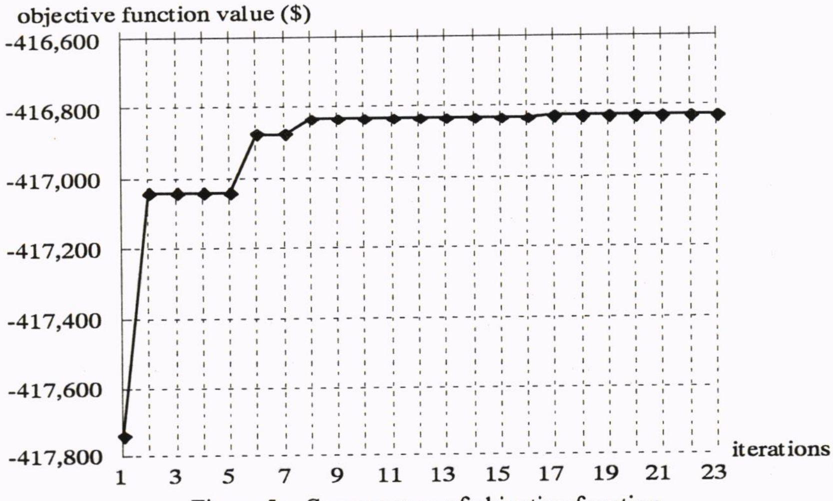

Figure 5 Convergence of objective function

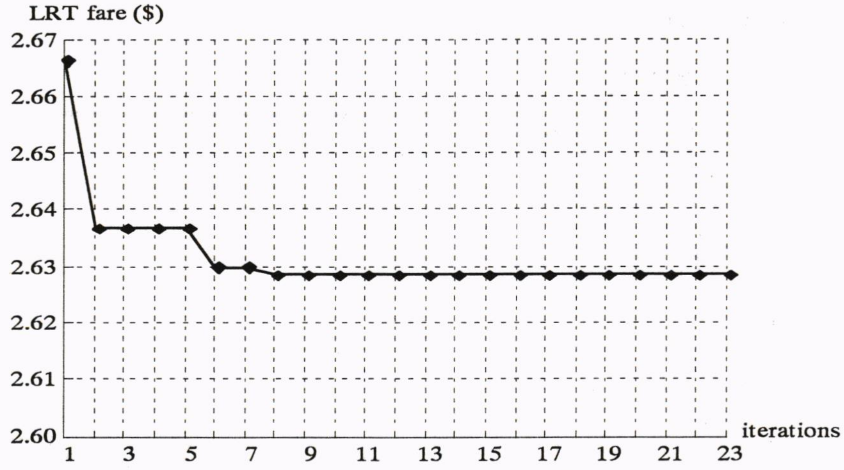

Figure 6 Changes of LRT fares with iterations

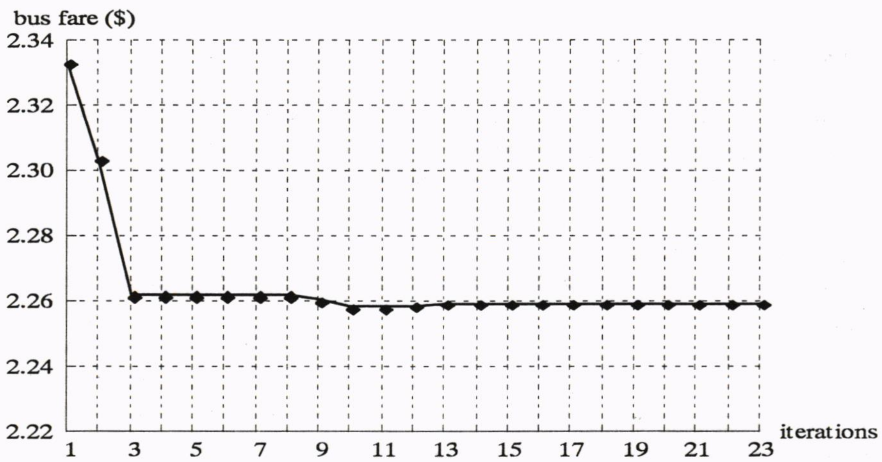

Figure 7 Changes of bus fares with iterations

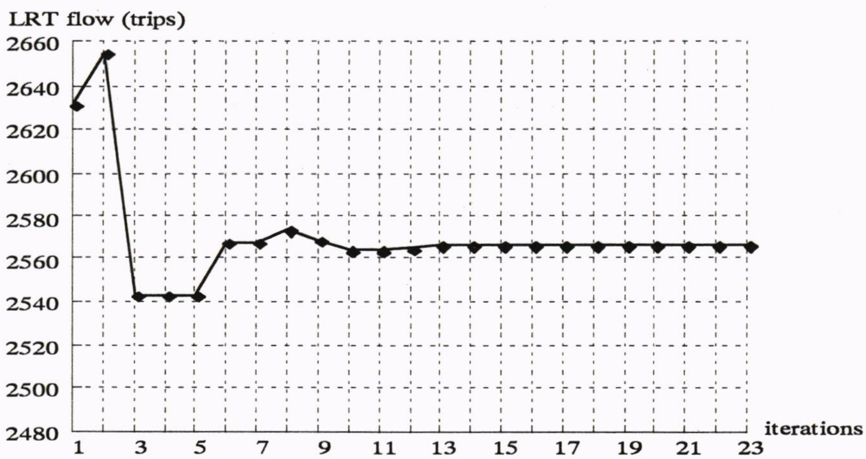

Figure 8 Changes of LRT flows with iterations

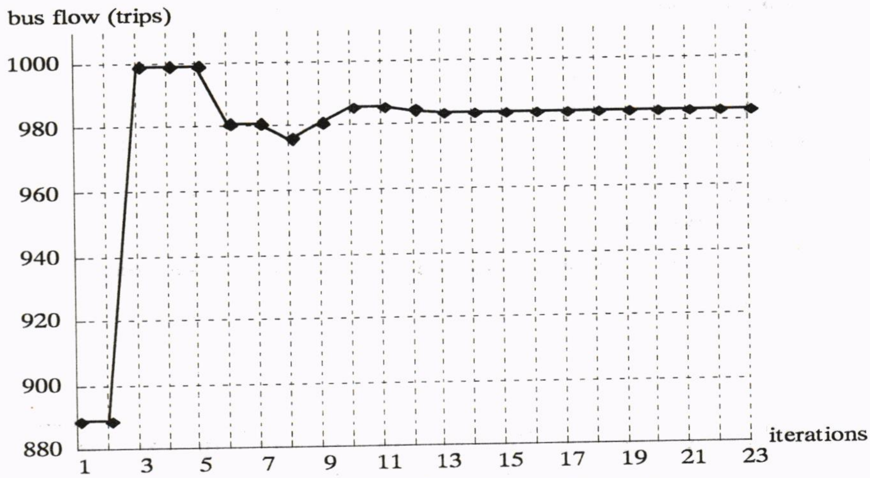

Figure 9 Changes of bus flows with iterations

4.3 Sensitivity Analysis of the Level of Subsidies

In this section we look into the effects of changing the level of subsidy rate on readership of public transits. These calculations are carried out using nonlinear sensitivity analysis for solving Stackelberg equilibrium. The results are shown in Fig. 10 to 14.

As are indicated in Fig.10 and 11, if subsidy rate is brought down, optimal transit fares would be increased so that the loads of transit user would increase. While transit users change their transportation mode to automobile, congestion on automobile network becomes more heavy. The same situation occurs in bus fare and trips as well. Thus, as is shown in Fig. 12 this effect spreads over OD pair 2-4 which consists of only automobile network. Judging from the above results, the level of subsidy and fare of public transit have to be decided in the contexts of the whole of the urban transportation system.

Fig. 12 shows that increasing subsidy makes the OD travel times smaller, which in turn implies that increasing subsidy rate results in the increase of social welfare. Unfortunately, the Ramsey equilibrium does not directly provide the optimal level of subsidy, however, we are able to find it by parameterizing the levels of subsidy as is conducted in Fig. 13. The Fig. 14 shows the cumulative change of social welfare and subsidy. This histogram is useful to explain the existing of the lower limit with respect to effective subsidy rate. In this example the effective subsidy rate is 50%.

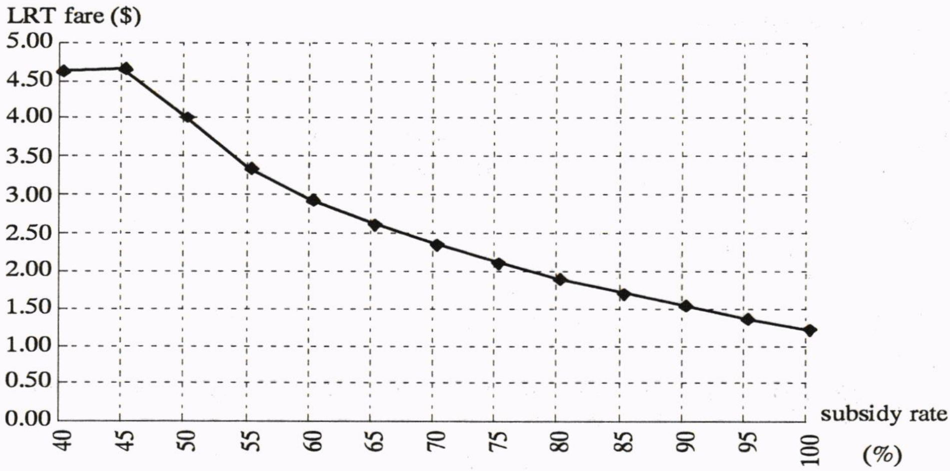

Figure 10 Change of LRT fares with different subsidy rate

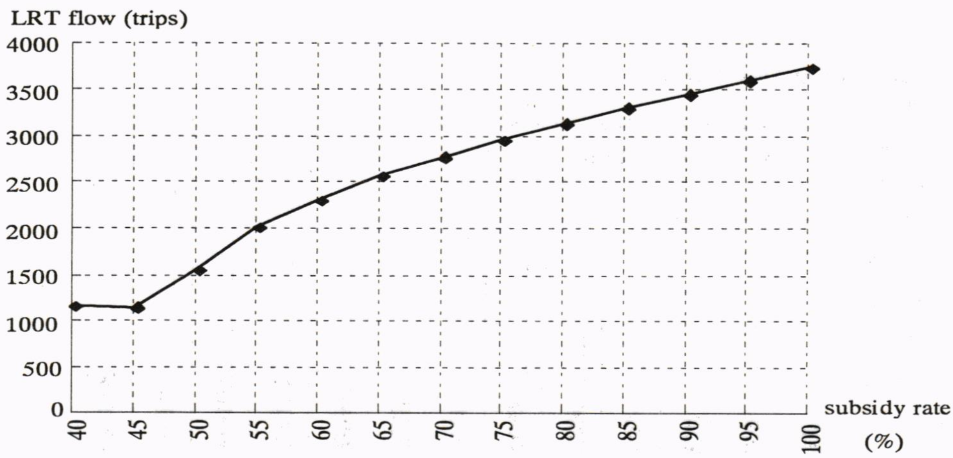

Figure 11 Change of LRT flow with different of subsidy rate

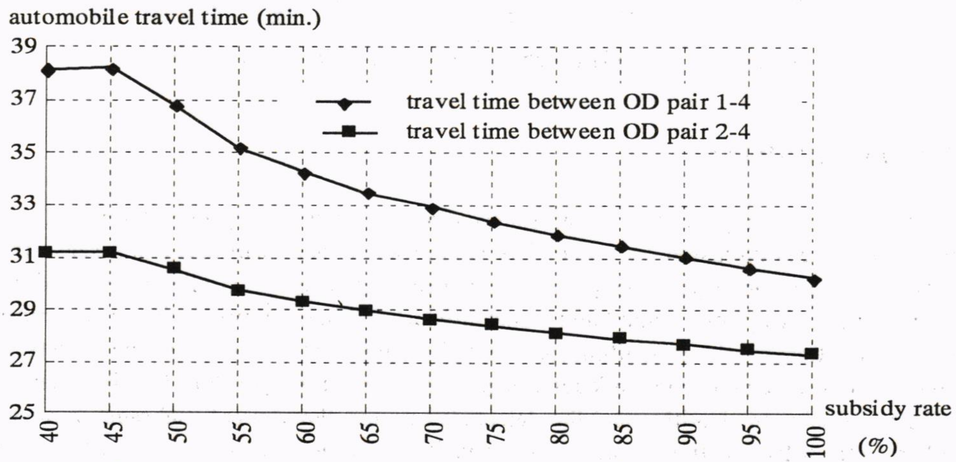

Figure 12 Change of automobile travel time with different subsidy rate

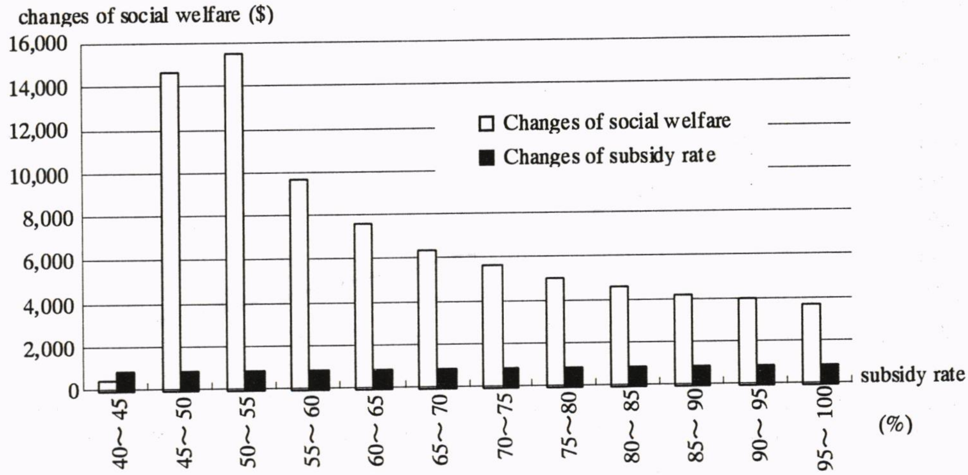

Figure 13 Change of social welfare with change of subsidy rate

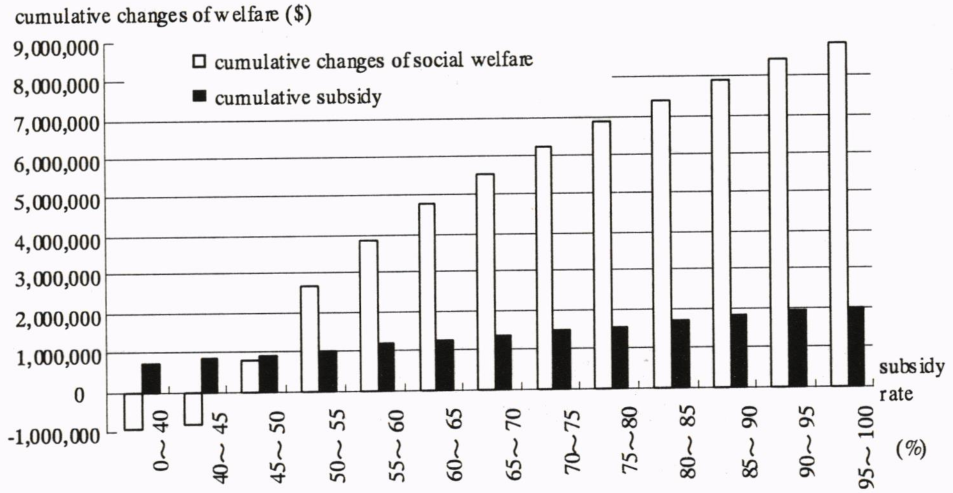

Figure 14 Cumulative changes of social welfare and subsidy

5. CONCLUSIONS

The conventional applications of the Ramsey rule to pricing of transportation services which have been discussed by economists are unrealistic and insufficient in a sense that network congestion effects on pricing have been neglected. On the other hand we proposed a Ramsey price equilibrium model in which the Ramsey price rule is restructured within the framework of multimodal network equilibrium to consider the effects of congestion of automobile network for pricing and its solution algorithm. In this paper the validity of solution algorithm and the deference between the conventional ramsey pricing rule for transit fares without considering network congestion and ramsey price equilibrium model are examined thorough the numerical example.

We apply the nonlinear sensitivity analysis which correspond to Stackelberg equilibrium to simplified urban transportation system. It is confirm that the method converges to at least local minimum. The effects for transit pricing spread whole of the urban transportation system through congestion on the automobile network. Therefore public transport operated under the zero profit and given a certain subsidy must set fares to maximize social welfare for whole of the urban transportation system. A ramsey price equilibrium model is useful this pricing. However the simplified system was treated in this paper, the solution method can be applied to real scale one except the cities which have complex transit network such as plural transit path flow vector could be defined with respect to an equilibrium link flows vector.

At end of this paper sensitivity analysis of subsidy rate is examined. If the upper level of prices are given in a significant way, the Ramsey equilibrium approach shows the minimum level of the government subsidy of achieve welfare maximization.

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