A NEW SIMULATION METHOD FOR DYNAMIC TRAFFIC FLOW

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abstract: The researches on Intelligent Transportation System in past decade have concluded that dynamic traffic flow theory is the foundation for developing ATMS/ATIS. Based on current literature the simulation-based methods are mostly feasible in practical applications, although they need some improvement. This paper, in order to capture more features of dynamic traffic flow and to improve the quality of simulation based solution, presents a new simulation procedure which calculates the time-dependent travel time according to the traffic density ahead the vehicle on the link but regardless the traffic characters behind the vehicle. Besides, an exact algorithm for network loading is developed which can calculate the time-dependent load of links and travel time simultaneously and accurately. Finally, computational experiments are reported.

1. INTRODUCTION

With the developments of Advanced Transportation Management System/Advanced Traveler Information System (ATMS/ATIS), the Dynamic Traffic Assignment (DTA) obtains increasing attention. To date, the work on DTA is mainly addressed at the extension of the static traffic assignment which is based on two type of objectives: User-Equilibrium(UE) and System Optimization(SO). As for UE, recent publications can be roughly classified into two classes: (a) analytical approach, including so-called optimal control theory approach and optimization approach classified by Jayakrishnan (1995). The main work includes: Merchant and Nemhauser (1978a,b), Carey (1986,1992), Wie et al. (1990,1994,1995), Friesz et al. (1989), Janson (1991,1992), Drissikaitouni (1992,1993), Ran, et al. (1993), Weymann et al. (1994) and Boyce, Ran, LeBlance (1995), Peeta and Mahmassani (1995), Lam and Huang (1995), ect.; (b) Simulation-based approach (Mahmassani and peeta, 1993; Jayakrishnan,1992; Ben-Akiv *et al*, 1994). Approach (a) possesses some important advantages for theoretical analysis, such as convergence and the optimality of solution. The cost for these advantages is the simplification of traffic behavior, particular the simplification of link cost function. The simplification usually leads the violation of first-in-first-out (FIFO) condition. Sometimes, excessive simplification will lose the traffic realism. Besides, this class of methods need to solve one or more very complicate mathematical models (much more decision variables and constraints) and does not ensure to get an assignment of integer flow. Comparatively, approach (b) captures the traffic dynamics better and results in an assignment of integer flow. But it is considered to lack of good feature in convergence and solution quality (Jayakrishnan, 1995). As for SO, recent works include Addison & Heydeker (1995), Ghali and Smith (1995), Mahmassani & Peeta (1995) and Ziliaskopoulos (1995), etc. These methods aim to find an assignment with minimum system cost and also need to solve very complicate mathematical models.

Studying the current literature we learn that simulation-based methods are mostly feasible for practical applications, particularly for on-line operations and decision making, although these methods need some improvement.

In this paper, in order to enhance the quality of simulation-based solution we propose a new speed function which is based on traffic density ahead the vehicle on the link but regardless the traffic characters behind the vehicle, and develop a new procedure for calculating the links' travel time in Section 2. This procedure can avoid the violation of FIFO condition. Furthermore, aiming to improve the accuracy of dynamic link load, based on given path for each O-D pair, Section 3 expresses an exact network loading algorithm, which yields an integer traffic assignment. The converging speed of this method is shown by computational experiments and their analysis of various scale of examples in Section 4.

2. TIME-DEPENDENT TRAVEL TIME

In this paper, i-j indicates an O-D demand with origin of i and destination of j, (i,j) indicates a link connects nodes i and j. A link means a connection of two nodes without side-exit and side-entrance. The cost of link (i,j) means the travel time from i to j.

An appropriate procedure for calculating the link cost is very important to dynamic traffic assignment (Daganzo, 1995a,b). Conventionally, $v_{ij}(l_{ij}^t)$ is used to denote the speed function on link (i,j) in the tth time interval, where l_{ij}^t is the traffic load—the number of vehicles presenting on the link during tth time interval. In this paper v_{ij} can be, for example, jumping, piecewise, etc.. On the other hand, v_{ij} is allowed to vary with time intervals. For example, the speed under the same load but in different time intervals (e.g. day and night) may be different. In this case we use v_{ij}^t to identify the time interval.

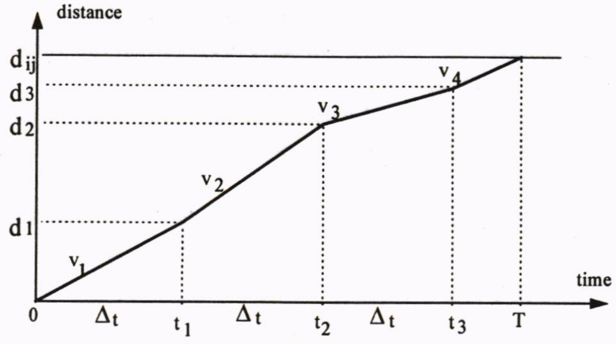


Fig.1. Existing Calculating procedure of time-dependent travel time

Travel time is obviously time-dependent because O-D demands are time-dependent. the existing simulation-based approach divides the travel time on one link into several parts according to time intervals, and calculate the running distance of every time interval according to the time-dependent loads of the link until the accumulative distance equals the link spatial length, Then, the accumulative travel time is the link's travel time during the corresponding time intervals. Fig.1 illustrates the calculating procedure, where d_{ij} *is* the spatial length of link *(i,j)*,*T* is the travel time of link *(i,j)*.

This approach captures the load's time-varying characters better than the analytical approach. Nevertheless, it does not account for the time-dependent location of the O-D demand (vehicles). Fig.2 shows two cases of loads in different time intervals on link *(i,j)*.

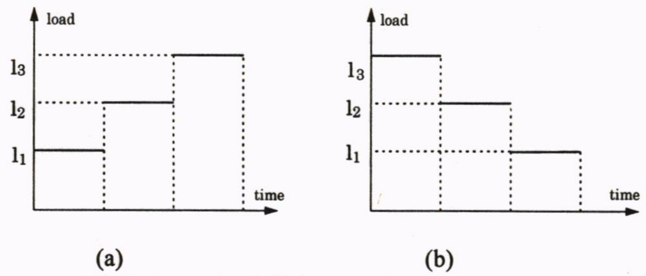


Fig.2. Loads in different time intervals

Fig.2 (a) indicates that the load increases during an O-D demand traveling on link *(i,j)* and Fig.2 (b) indicates the opposite. By the existing approach, the travel time of above two cases would be the same. But by the traffic realism the travel time of case (a) should be less than that of case (b) because the variation of load behind a traveler will scarcely affect his/her travel time.

Based on above idea we propose a simulation-based method of calculating travel time as follows: Let

$$v(N_s^t, s_t) = v_0 - \beta \left[\frac{(N_s^t)^{\alpha_1}}{(s_t)^{\alpha_2}}\right] \tag{1}$$

where, $v(\cdot)$= speed function. To help the method get out off the jam state we define a minimum speed $v_{\min}$. That is, we set $v = v_{\min}$ when the value of v given by Eg.(1) is less than $v_{\min}$.

s_t= the distance between current location of the considered vehicle and the node the vehicle is moving toward. Afterwards, we sometimes use s_t to specify vehicle's locations.

N_s^t= the number of vehicles on link *(i,j)* ahead the considered vehicle in the *t*-th time interval when the vehicle is at the location of s_t.

v_0 = the free flow speed.

$\alpha_1, \alpha_2, \beta$ = parameters.

Eg.(1) is consistent with the known relationship between speed and density shown in Fig.3. By selecting appropriate values of $\alpha_1, \alpha_2, \beta$ we can capture the traffic realism well. There is not any restriction to selecting values of these parameters in this paper.

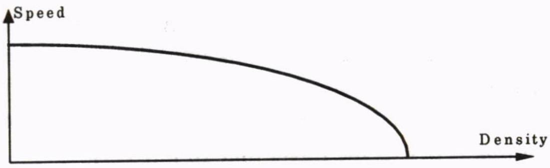


Fig.3. Relationship between speed and density

Suppose the considered vehicle enters node i in the t_0-th time interval, then, in the t-th time interval, the vehicle number ahead the consider vehicle is (it illustrated in Fig.4):

$$N_s^t = l_{ij}^t - \sum_{k=t_0+1}^{t} I_{ij}^k \tag{2}$$

where, I_{ij}^k= the number of vehicles entering link *(i,j)* during the *k*th time interval.

l_{ij}^t = the load (total vehicle number) in link *(i,j)* during *t*-th time interval.

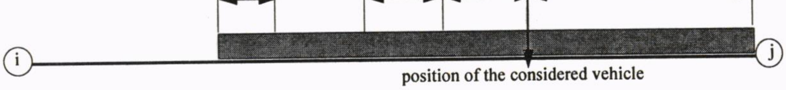


Fig.4. Vehicle number ahead the considered vehicle

In order to avoid the violation of FIFO condition we propose a new procedure for calculating travel time. As shown in Fig.5 we use spatial interval to replace time interval shown in Fig.1. In each step the vehicle moves a distance of Δd ($\Delta d \le \Delta t \cdot v_{\min}$, Δt is the span of time interval).

Suppose the link is divided into q spatial intervals and a remainder Δr ,then the travel time of link *(i,j)* can be expressed as

$$C(q,\Delta r) = \sum_{l=1}^{q} \frac{\Delta d}{v(N_s^{t_l}, s_l)} + \frac{\Delta r}{v(N_s^{t_{q+1}}, s_{q+1})}\ , \tag{3}$$

where, $s_l = d_{ij} - (l-1)\Delta d$, t_l is the corresponding time interval when the vehicle is at the location of s_l.

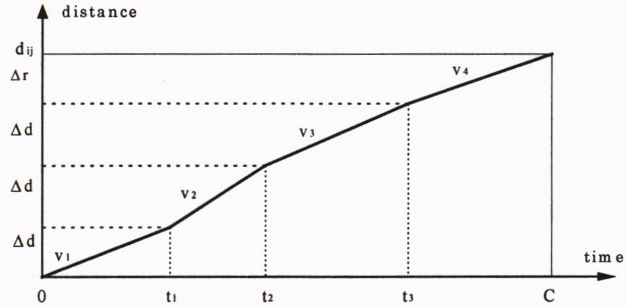


Fig. 5. New procedure of calculating travel time

It should be clear that the new procedure for calculating travel time complies with the FIFO rule, and allows the consideration of queuing at conjunctions.

It is necessary to point out that the method for calculating the travel time does not affect the method for δ-Equilibrium assignment presented in next section.

3. AN EXACT NETWORK LOADING ALGORITHM

As we have seen in section 2 the time-dependent load of links is the foundation for calculating the time-dependent travel time. Nevertheless, as we can easily image, the load of links is entirely dependent on the time intervals of vehicles pass the links, while this is actually the time-dependent travel time. So, how can we calculate the time-dependent load of link and the travel time simultaneously and accurately? This section will express such an algorithm.

For the purpose of conciseness we use one subscript to replace the two subscripts and the up-script of O-D demand a_{ij}^k, and re-label the O-D demands by $a_1, a_2, \ldots, a_n$.

For a given route set for all O-D demands, let

r_k^l = the label of the node following node l on the route for a_k, including the origin (indicated by r_k^0) and the destination.

(o_k, p_k) = the link that a_k is running on currently.

s_k = the distance from the current location of a_k to node p_k. The meanings of p_k and s_k are shown in Fig.6.

C_k = the accumulative travel time of a_k.

t_k = the time of a_k entering the network.

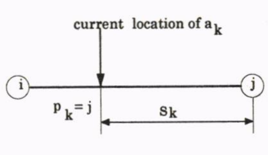
T_k = the time interval that t_k belongs to.

D_k = the destination of a_k.

Δt =the span of time intervals. For the sake of convenience we use the same interval length here although our method allows different lengths.

d_{ij} = the spatial length of link (i,j).

$l_{ij}^t, I_{ij}^t, \Delta d$ = the same meanings respectively as in Section 2.



Fig, 6. Dynamics of vehicle

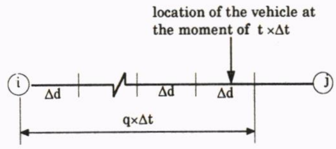


Fig.7. Moving steps of vehicle

<u>Main Idea</u>: Start with the first time interval, let every vehicle (O-D demand) currently on the network move forward q spatial intervals such that the vehicle just enters the next time interval. This idea is illustrated in Fig. 7. Update the time-dependent loads, travel time, vehicle's location ect., step by the next time interval and repeat above procedure until all vehicles reach their destinations.

In detail, the network loading algorithm can be expressed as follows:

1°. $t \leftarrow 1, C_k = 0 (\forall k)$.

2°. For any a_k such that $T_k = t$, let $i = r_k^0, j = r_k^i$, and set

$$I_{ij}^t \leftarrow I_{ij}^t + a_k,\ l_{ij}^t \leftarrow l_{ij}^t + a_k,\ o_k = i,\ p_k = j,\ s_k = d_{ij}.$$

3°. If $s_k = 0$ holds for any $a_k (T_k \le t)$, then, stop (the loading procedure ends); otherwise, select one a_k such that $s_k > 0$, go to 4°.

4°. Let $i = o_k, j = p_k$. Determine the move steps q such that

$$t_k + C_k + \Delta_{q-1} < \Delta t \cdot t \le t_k + C_k + \Delta_q ,$$

where, $\Delta_q = \sum_{r=1}^{q} \frac{\Delta d}{v(N_s^{t_r}, s_k^r)}, s_k^r = s_k^{r-1} - \Delta d, s_k^0 = s_k, N_s^{t_r}$ is given by Eg.(2) with t_r indicating the time interval that $(t_k + C_k + \Delta_r)$ belongs to.

If $T_k = t$ go to 5°; otherwise go to 6°.

5°. Set

$$s_k \leftarrow s_k - q \cdot \Delta d,$$
$$l_{ij}^{t+1} \leftarrow l_{ij}^{t+1} + a_k,\quad C_k \leftarrow C_k + \Delta_q.$$

Go to 7°.

6°. i) If $q \cdot \Delta d < s_k$, then, return to 5°.

ii) If $q \cdot \Delta d \ge s_k$ and $p_k \ne D_k$, let

$$o_k = p_k,\ p_k \leftarrow r_k^{p_k},\ s_k = d_{o_k, p_k},$$
$$C_k \leftarrow C_k + C(\bar{q}, \Delta r),\ l_{o_k,p_k}^t \leftarrow l_{o_k,p_k}^t + a_k,\ I_{o_k,p_k}^t \leftarrow I_{o_k,p_k}^t + a_k,$$

where, $C(\bar{q}, \Delta r)$ is given by Eg.(3) with $\bar{q}$ being the rest spatial steps from the vehicle's current location to node p_k.

Return to 5°.

iii) If $q \cdot \Delta d \geq s_k$ and $p_k = D_k$, then,

$$s_k = 0, \quad C_k \leftarrow C_k + C(\bar{q}, \Delta r).$$

Go to 7°.

7°. Check if all a_k such that $p_k \leq t$ and $s_k \neq 0$ have been treated. If yes, $t \leftarrow t+1$, return to 2°; otherwise, select one untreated a_k and return to 4°.

4. PERFORMANCE OF THE METHOD

For purpose of test we set $\alpha_1 = \alpha_2 = 1$ throughout this section.

Fig.8 illustrates the network structure of the test problems, where every edge indicates two arcs of two directions. For all test examples a_{ij}^k and d_{ij} are generated randomly such that $a_{ij}^k \in \{1,2\}, d_{ij} \in [10,20]$. A statistics of some computational experiments are given in Tab.1. From the CPU we can see that the method is quick enough for on-line applications.

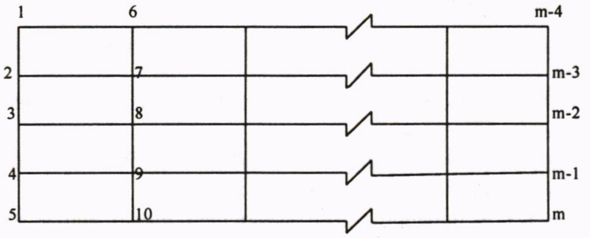


Fig.8. Network structure of test problems

Tab.1. Statistics of some computational experiments

m	Nl	p	n	v_0	β	Δt	CPU
50	170	50	17491	8.0	0.2	0.5	3.74
50	170	100	18048	8.0	0.2	1.0	4.39
50	170	200	25052	8.0	0.2	1.0	6.43
60	206	60	21171	8.0	0.2	0.5	7.36
100	350	100	25073	8.0	0.2	0.5	10.40
100	350	100	32120	8.0	0.2	1.0	8.94
100	350	200	35586	8.0	0.2	0.5	11.37
100	350	300	28505	8.0	0.2	0.5	9.84
100	350	200	40997	8.0	0.2	1.0	10.46
100	350	300	49741	8.0	0.2	1.0	12.81
120	422	300	47094	8.0	0.2	1.0	14.63
150	530	200	51600	8.0	0.2	1.0	18.37
150	530	300	68458	8.0	0.2	1.0	28.93
200	710	200	64705	8.0	0.2	1.0	33.25

where, m= the number of nodes in the network.

N_l= the number of links.

p= the number of time intervals. These are only the time intervals during which O-D demands generate, the total time interval (including those during which vehicles are running) will exceed this number.

n= the number of O-D demands.

v_0 = the free flow speed.

β= the speed parameter.

CPU= the running time (seconds) on DEC-Tubolaser/AS8400.

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