A SIMULATION ANALOGY FOR UNTIDY MULTI-LANE UNIDIRECTIONAL TRAFFIC FLOWS

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abstract: Analysis of two dimensional vehicular interactions acquires special importance in highway design and operation. Literature on representations of multi-lane traffic behaviour, which basically averaged the stream characteristics of individual lanes over the cross-section of the carriageway with simple lane changing manoeuvres, is reviewed leading to the conclusion that earlier models all assumed that every vehicle travels in the middle of the lane. However, existing models may produce inaccurate results where lane discipline is poor and disorderly flowing traffic is predominant. This paper introduces a new analogy through which the effect of untidy flow situations on throughput is expressed by means of both the accommodation and the overtaking problems.

1. INTRODUCTION

In addition to the longitudinal analysis of traffic flow, a number of studies dealt with lateral characteristics of multi-lane roadways. In the following section, the literature on the lateral aspect of traffic flow is briefly reviewed, followed by a summary of the main findings of this previous work. Section 3 and 4 captures and emphasises problems identified in earlier studies, and introduces the problem of untidiness. Section 5 presents the approach of the paper to this problem. Finally, a conclusion and the authors' ongoing research are given in Section 6.

Although there are different ways of naming traffic lanes, like left/right, offside, nearside, etc., in this study lanes are defined as shown in Figure 1. Traffic, in all figures of the paper, is unidirectional and based on/converted to the British driving system.

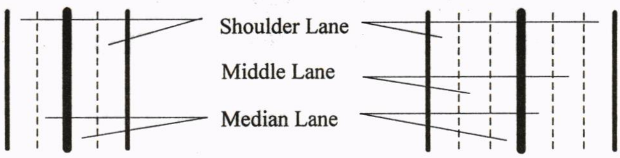


Figure 1: Lane naming system used throughout the paper for two and three lane highways.

2. BEHAVIOUR OF MULTI-LANE TRAFFIC FLOW

2.1 Introduction

With increasing road traffic, widely used multi-lane roadway facilities play an important role in transportation. Lateral analysis of the vehicular behaviours on multi-lane highways goes back to the early sixties. The previous research in this field can basically be classified into two groups as: lane changing and lateral distribution of vehicles.

2.2 Lane Changing

Lane changing is a very important and complex phenomenon in highway traffic flow. There are variety of reasons why drivers change lanes. Most of the researchers have studied lane changing in a macroscopic fashion by averaging drivers' behaviour, since to model lane changing decisions mathematically, taking into account all causes for a lane change, would be difficult.

On a multi-lane highway during relatively congested conditions, the common tendency of drivers is to move to the fast moving lane from the slow moving one. These switches are defined by Gazis, D. C. *et al.* (1962) as "density oscillations between lanes". Therefore one can now talk about different flow characteristics among the lanes having a dynamic nature (i.e. continually changing in time). The study (Gazis, ibid.), after introducing the term "equilibrium density distribution", states that the rate of change of vehicles between two neighbouring lanes is proportional to the difference of the deviations of their densities from their equilibrium values. Mathematically, for a two-lane uniform unidirectional highway, the rate of lane changes was hypothesised to be given by:

$$\frac{\partial q_1}{\partial x}+\frac{\partial K_1}{\partial t}=a\,(K_2-K_1)\ ,\qquad \frac{\partial q_2}{\partial x}+\frac{\partial K_2}{\partial t}=a\,(K_1-K_2) \tag{1}$$

where K_i and q_i are deviation from the equilibrium density and the flow in lane i respectively, and a is a positive constant with dimension time^{-1}. They also concluded that the danger of instability increases with the number of lanes. In addition to this linear formulation, Oliver and Lam (1965) proposed a non-linear model, in which traffic was assumed to behave as a compressible fluid, obeying the equation of continuity. For lane changing on a two lane unidirectional roadway:

$$\frac{\partial k_1}{\partial t}+\frac{\partial q_1}{\partial x}=P_{21}(x,t)-P_{12}(x,t) \quad , \quad \frac{\partial k_2}{\partial t}+\frac{\partial q_2}{\partial x}=P_{12}(x,t)-P_{21}(x,t) \qquad (2)$$

where

k_i is the density of lane i;
q_i is the flow of lane i;
$P_{12}(x, t)$ is the lane change function (the transfer of vehicles from lane 1 to lane 2); and
$P_{21}(x, t)$ is the lane change function (the transfer of vehicles from lane 2 to lane 1).

In order to be able to implement and interpret such multi-lane traffic engineering studies, the measuring of lane change becomes an important issue. Worrall and Bullen (1970) introduced two terms: "frequency", the number of lane changes occurring among all lanes along a given length of road and over a time span; and "pattern", the distribution of lane changes between specific lane-lane pairs along a given road length and over a time span. Based on this research, the same authors in another study (Worrall, R. D. *et al.*, 1970) reported an elementary statistical macroscopic model by treating lane changing on a multi-lane highway as a stochastic phenomenon.

In another piece of research, the basic lane changing hypotheses available at that time, mentioned above, were validated and compared experimentally from aerial data (Munjal and Hsu, 1973). They concluded that statistically the non-linear model of Oliver and Lam produced quite satisfactory results for every lane of a three-lane motorway, while the other two (the approaches of Gazis and Worrall) do not. Therefore the models, generally speaking, were ranked as the non-linear model by Oliver and Lam, the stochastic model by Worrall, Bullen and Gur, and the linear model by Gazis, Herman, and Weiss.

Sparmann (1979) basically stated that lane changing to the median lane requires considerable concentration and attention, as the driver is forced to observe traffic flow around him, while lane changing to the shoulder lane is easy to determine. Lane changings are disadvantageous for traffic safety and driving comfort in some cases when a lane changing involves a deceleration exceeding 1.25 m/s^2. When evaluating the accident rate with reference to traffic flow, the conflicts arising from lane changings and their frequency should be taken into account. Later McNees (1982) concluded the following:

- as traffic flow increased, manoeuvring distance also increased,
- the time required to complete the manoeuvre increased as the traffic flow increased, and
- as the number of lanes increased, the manoeuvring distance also increased, therefore the sign-placement distance should take into account the maximum number of lane changes.

One of the first multi-lane microscopic traffic flow simulators that include lane changing was introduced by Gipps and Wilson (1980). The package, MULTSIM, was capable of governing the movement of a vehicle within its present lane, and of regulating its ability and tendency to change lanes. He later expanded this by implementing a behavioural approach (Gipps, 1986). He proposed a structure to connect the decisions which a driver has to make before changing lanes. The only weak point of the research might be that his model was not validated due to its complexity - it can only be invalidated by finding a fault. Similarly, Ferrari (1989) revealed that drivers on short trips, like those travelling on urban motorways, behave in a different fashion, insofar as the time spent in waiting to overtake a slower vehicle is of little importance. They do not have a preferred lane, and thus elect a choice which, being characterised by a more limited number of lane changes, results in safer and less stressful trips.

Finally, Yousif and Hunt (1995) developed a microscopic simulation program that models lane changing behaviour on multi-lane unidirectional highways of the UK, including the relationship between lane changing frequency and flow. They compared their findings with some other countries as seen in Figure 5.

2.3 Lateral Distribution of Vehicles

This part of Section 2 focuses on different characteristics of each lane of a multi-lane unidirectional highway and consequently the reasons behind lane switches. For example, Munjal and Pipes (1971) investigated propagation of density perturbations in the vicinity of on-ramps. It is known that joining from an on-ramp to the main road causes sudden increases in density of the shoulder lane. As the distance from this entry point increases downstream, the traffic density in the shoulder lane decreases with diminishing perturbation. This leads to relatively higher lane changing manoeuvres in the on-ramp region compared to other parts of the roadway. The main conclusion of their study, based on the mathematical continuity model of an idealised "traffic fluid", which was calibrated using real data for a three-lane roadway, is demonstrated in Figure 2.

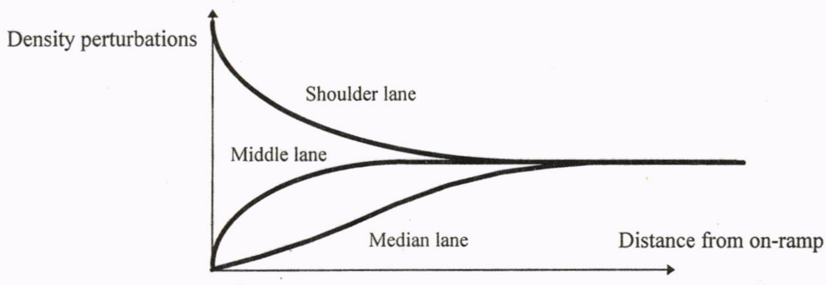


Figure 2: Relative propagation of on-ramp density perturbations in different lanes (from Munjal and Pipes, 1971).

Pahl (1972) was interested in special operational and safety problems in the vicinity of an off-ramp where vehicles change their lanes to a slower lane so as to leave the highway. Results of the study showed that through vehicles tend to move away from the lane adjacent to the off-ramp at a considerably distance upstream from the ramp nose and to return to that lane as soon as the exiting traffic has departed. Similarly, Wemple, E.A *et al.* (1991) developed a motorway lane model which predicts flows by lane in the vicinity of ramp junctions and major weaving sections. One of the results of the empirical observations indicated that when lane changing falls below approximately 6% of the PCU's/hr in the lane being studied, the motorway section may be considered as a straight pipe section - away from ramps, junctions or weaving areas. Lane distribution vs. flow data indicated that under high flow conditions the median lane is the primary carrier, and the shoulder lane carries the smallest percentage of traffic.

Distribution of vehicles across the lanes is an important factor determining traffic flow characteristics in multi-lane highways. Most of the research in the field of traffic flow, in general, was based on single lane stream analyses. They either combined the characteristics of each lane or considered average behaviour over the lanes, in order to represent the traffic flow as a whole. However, some research, undertaken in various countries, Israel, the USA, Canada, Germany and the UK (presented in this order in the following), emphasised distinct characteristics of each lane across the carriageway, and the interaction between them.

Mahalel and Hakkret (1983) revealed interesting and useful evaluations stating that the arrival patterns of vehicles in a lane of a multi-lane unidirectional highway is dependent on the arrival patterns of vehicles in the other lanes in the same direction, and this feature necessitates the use of models which simultaneously describe vehicle arrivals on the roadway. Their model was based on the unconditional probability of vehicle arrivals in each lane, tested on Israel motorways. Concerning highway safety, they also said that relatively high usage of median lane as well as a transfer of vehicles from the shoulder lane to the median lane, as traffic flow increases, may have a significant negative impact. They also confirmed the behaviour of multi-lane traffic flow which fits the Markov process, proposed earlier by Worrall, R. D. *et al.* (1970) and Rorbech (1974).

Chen (1986), implementing lane by lane speed-flow studies on a six-lane motorway in the USA, concluded that the shoulder lane suggests quite a linear speed-flow relationship, while middle lane appears to be a second order curve; and median lane is composed of two straight lines intersecting at the flow level of 1720 vehicles/h/lane. His work also revealed that the speed of the median lane is affected by speed of the middle lane; and speed of the shoulder lane is dependent on total flow rather than lane flow. Hall and Gunter (1986), in like manner, studied flow-occupancy relationships on a three-lane Canadian motorway, concluding that discontinuous relationships, suggested by most of the earlier studies, as demonstrated in Figure 3, are not necessary to describe multi-lane flow-density relationships. Hall and Lam (1988) supported this, stating that as the congestion moves upstream from a bottleneck, flow rates in the individual lanes change. Their study resulted in different flow-density shapes for each lane. A smoother, more rounded and more continuous form of the flow-density curve of the shoulder lane than for the middle and median lanes was explained, by the authors, by lane changing behaviour as drivers attempt to take advantage of the last opportunity for uncongested operation in adjacent lane(s).

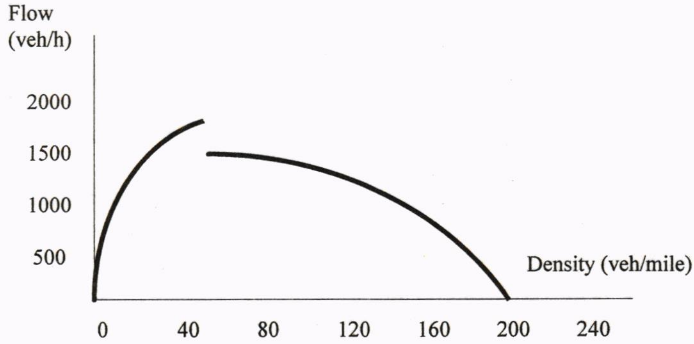


Figure 3: A typical-flow density diagram, with a drop around the capacity, suggested by most of the researchers (from Hall, F. L. *et al.*, 1986).

Heidemann (1994) developed a model, which was tested with the data collected near the city of Karlsruhe, Germany, to describe the distribution of vehicles to the individual lanes. He argued that if a more balanced lane utilisation were to be achieved, for larger traffic flows, the increase in capacity and the decrease in traffic congestion would be considerable. His interim result was that the proportion of vehicles travelling in the shoulder lane of a two-lane unidirectional roadway is equal to the probability of a vehicle in the median lane making a change to the shoulder lane. However, this probability decreases with increasing traffic flow since increasingly fewer headways of sufficient length are available for changing to the shoulder lane. Figure 4 shows the relationship between the number of vehicles in each lane and traffic flow.

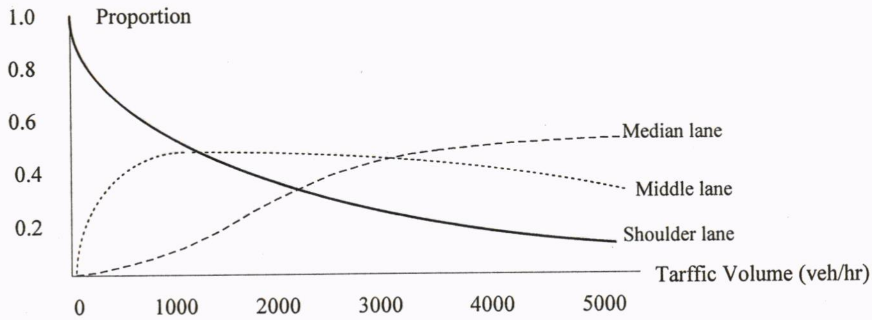


Figure 4: Proportion of traffic flow on the individual lanes of a three-lane unidirectional roadway (from Heidemann, 1994).

Yousif and Hunt (1995) suggested an empirical model, based on the data collected under different traffic flow conditions in the UK. They concluded that drivers' lane choice is one of the parameters which determines the lane utilisation and lane changing behaviour. It is extremely difficult to obtain data on how and why drivers choose their lanes. The behaviour could be affected by different speed limits and different traffic regulations. Their investigations produced quite similar relationships between flow and vehicular distribution to that of Heidemann's study shown in Figure 4.

2.4 Substantial Findings of the Previous Work

In the light of the literature reviewed in Sections 2.2 and 2.3 the main characteristics of multi-lane traffic flow can be summarised as follows. This paper's contribution will be presented in later sections, after some new terms and concepts are introduced.

i. Lane changings, and consequent safety considerations, have importance when judging the quality of traffic flow.
ii. There is a proven relationship between lane changing frequency and traffic flow, and this relationship exhibits different forms in different places (as seen in Figure 5).
iii. Automation in highways improves lane changings and therefore better flow-density relationships can be achieved in aggregate.
iv. Most of the previous work considered steady state situations. Investigation of forced flow or jammed circumstances is still a weak point.
v. Each lane has its own characteristics, and they interact with each other.

vi. Density oscillations between the lanes are observed especially around the ramp junctions.
vii. Main factors affecting lateral distribution of traffic are: driver behaviour and attitude; traffic flow; highway geometry and layout; and road markings.
viii. In general, with increasing flow, a decrease in percentage of vehicles in the shoulder lane and an increase in percentage of vehicles in the median lane are generally observed as seen from Figures 2 and 4.

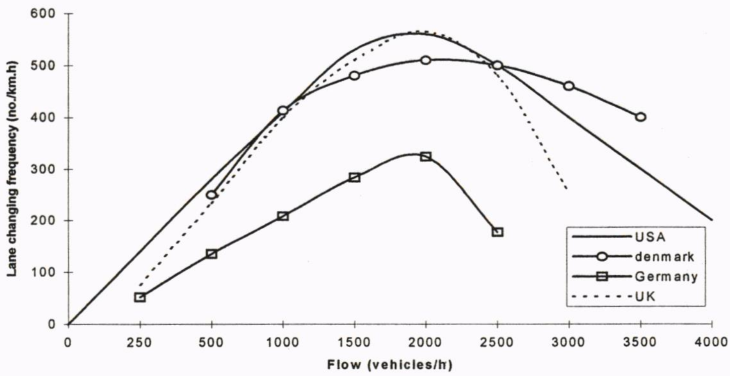


Figure 5: Approximate trends of the relationship between flow and lane changing frequency observed in different countries (from Yousif and Hunt, 1995).

3. INTRODUCING UNTIDINESS IN MULTI-LANE TRAFFIC FLOW

It is believed that the term "untidy traffic flow" and other related expressions, such as Degree of Untidiness, Lateral Discomfort, and etc., are first used by this research in the context of traffic engineering. Untidy traffic flow refers to the situation of both low lane discipline and disorderly traffic flow observed on a multi-lane unidirectional highway. In other words, the number of vehicles that do not travel, during a long period of time, within the boundaries of a single lane is considerably high in untidy flow situations. The order of "long period of time" is appreciably longer than the time that is needed by a vehicle in an overtaking manoeuvre. For example the depiction given in Figure 6(a) is a typical untidy traffic flow situation on a three-lane motorway.

The reason why such a circumstance of traffic flow has not received much attention by researchers could be that this type of traffic behaviour is very unlikely to occur in almost all developed countries. However, in most of the developing countries tidiness of traffic flow is fairly poor. Main reasons behind this circumstance may be explained by some or all of the following:

i) poor driving discipline: infringement of lane lines and impatient attitudes (trying to make quick use of free spaces on the roadway),
ii) budgetary problems of responsible highway authorities: lack of regular renewals of road markings and lack of existence of cats-eyes between lanes, and
iii) imperfection in highway design: insufficient length of weaving areas and badly designed exit and entry points.

"Degree of (Un)tidiness", another term that needs now to be defined, is the *lateral position of vehicles* across the carriageway. This is slightly different from the *lateral distribution of vehicles* over the lanes, which was examined by various researchers, presented in Section 2.3. In those studies, the number of vehicles that appears in each lane was the unit of that type of distribution. They, no matter at what horizontal location of a lane a vehicle is, all assumed that as long as that vehicle happens to be within the lane, it was considered to be travelling in the centre of the lane. However, this could be quite a rough assumption in untidy flow situations, and may cause inaccuracies in traffic flow analyses. Therefore Degree of Untidiness, used in this paper, is the skewness of the curve on which each point represents the number of vehicles (ordinate) as well as their lateral position (abscissa), as seen in Figure 6(b). Characteristics and determinants of Degree of Untidiness, such as skewness, peak and minimum points, longitudinal pattern of the distribution along the highway, relationship between the shape of the diagram and the fundamental characteristics of traffic flow, dynamic behaviour of the shape in time, statistical features of the distribution and etc., will later be published in a separate paper.

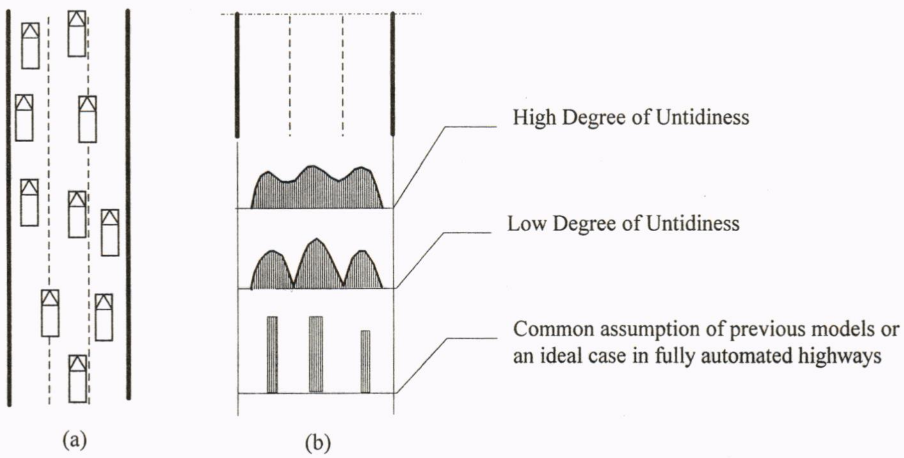


Figure 6: A typical untidy multi-lane traffic flow (a), and the distributions of lateral positions of vehicles (b).

4. IMPORTANCE OF RESEARCH ON UNTIDY TRAFFIC FLOW

As stated in Section 2, common tendency in previous research was to establish traffic flow models on the basis of the assumption that all vehicles travel in the centre of lanes unless they overtake or change lane. This approach, of course, does not affect much the accuracy of results in tidy flows. Nevertheless, considering the situation given in Figure 7, if the tidy flow, (b), is the simplification of the untidy flow, (a), and if the case of untidiness, (a), is observed many times within a certain distance or time, then the representation of the traffic flow of this approach becomes relatively weak. This weakness is the result of the influence of vehicle i on vehicle k and the high frequency of occurrence of this type of interaction among disorderly flowing vehicles.

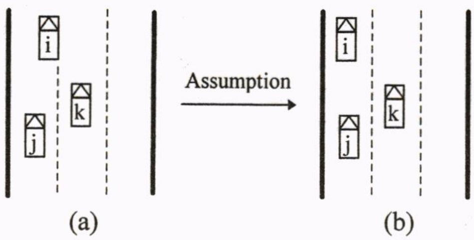


Figure 7: Common assumption made for model simplifications in tidy flows.

Although the microscopic fashion of these interactions will be discussed in detail in the next section, generally speaking, it is believed, at this stage of the research, that there is a relationship between the interactions of vehicles and the capacity of highways. Figure 8 shows the rough estimation of this relationship, which ongoing work is expected to clarify. With increasing untidiness in traffic stream, the throughput is likely to decrease down to a certain level, after which an extra (artificial) lane is built up by drivers - and this may or may not cause some increase in traffic flow. Although this case (with an additional lane) seems to have tidier flows in each lines of vehicles, an illegal lane formation is still an indication of abnormality and will be considered as untidy in this work.

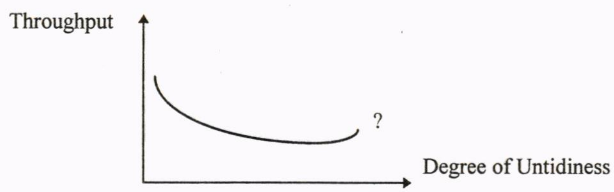


Figure 8: Prediction of a general relationship between throughput and untidiness.

Furthermore, untidiness has another effect on capacity, as far as long-term throughput is concerned (i.e. weekly, monthly or annual volumes). This effect is based on the idea that untidiness is potential source of lack of safety in traffic flow. Disorderliness and high frequency of lane changes may increase accidents, and consequently lane closures, bottlenecks, etc. and result in capacity drops in long term. Sparmann (1979) confirms this idea (see Section 2.2).

5. PHILOSOPHY OF THE APPROACH

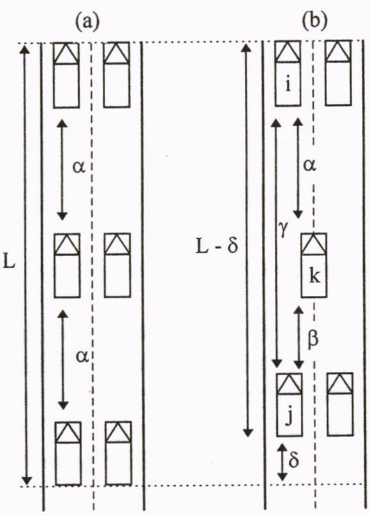
5.1 Introduction

Since the main concern of the research is to investigate the relationship between traffic flow and untidiness, the effects of disorderliness on the determinants (components) of traffic flow have first to be understood. The fundamental equation of traffic flow theory states that the flow rate is equal to the products of the speed of the stream and the vehicular density: flow = speed × density. As a mater of fact, untidiness to some extent reduces the density of traffic. As far as operations before capacity (steady-state conditions) are concerned, reduction in density causes decreases in flow. Therefore untidiness has an impact of flow reduction. The hypothesis given in Figure 8 assumes that after a certain level of untidiness (i.e. after the creation of an additional artificial lane) some increase in flow rate might be expected, although this forced and unsteady situation of the flow with an extra illegal lane is outside the interest of the present research. Let us now explore some various possibilities of vehicular interactions in multi-lane untidy traffic flow through the issues of "accommodation" and "slowdown".

5.2 Issue of Accommodation

In a very simple form of untidiness, Figure 9(b), all vehicles are assumed to be travelling at the same constant speed in both cases (a) and (b). Near capacity operation, due to the untidiness in Case (b) the number of vehicles accommodated in the same length of the road is one less, compared to Case (a), leading to a density drop on one hand. Closer following distance 'β' on the other hand, increases the density (since density=1/spacing). The problem now becomes the issue of which of the following two statements is correct:

i) If density (a) > density (b), then it can be concluded that untidiness decreases density.
ii) If density (a) < density (b), then it can be concluded that untidiness increases density.



α : ordinary Car Following distance
β : secondary Car Following distance (between j and k)
γ: remote Car Following distance (between i and j)

$\beta < \alpha$
$\delta = \alpha - \beta$
$\gamma = \alpha + \beta +$ length of vehicle k

density (a) = 6 vehicles / L
density (b) = 5 vehicles / L - δ

Figure 9: Demonstration of secondary Car Following distance, β.

In other words, all needs to be performed is to find out what the 'β' distance is. Distance 'α' is the minimum safe following headway easily expressed by ordinary Car Following theories, whereas determination of 'β' is more complex and difficult. Almost all of the previous research concerned with the Car Following approach examined single lane vehicular flows assuming low degrees of untidiness with no lane changings. Treatment of multi-lane traffic flow as a whole, however, was based on the averaging analyses of individual lanes over the carriageway, and lane changes between them, as discussed in the literature review section of the paper.

Distance 'β' is what the authors call "the staggered following distance of a secondary Car Following case", since the interaction between the disorderly travelling leading vehicle and the following vehicle is an additional follow-the-leader case to the existing one, taking place through the distance 'γ' (i.e. vehicle j has two leaders to follow by two different distances, 'β' and 'γ'). Determination of 'β' is one of the parts of the authors' ongoing research, to be published in a further paper. Only the theoretical approach is explained here through the concept of "Lateral Discomfort":

Existence of other vehicles in a roadway causes discomfort to some others. This discomfort can be classified as longitudinal and lateral. Whilst Longitudinal Discomfort has been studied by many researchers by means of the Car Following theory, Lateral Discomfort, however, has not been investigated properly because tidiness is predominant in most of the developed countries and, of course, standardised lane widths of motorways are sufficiently wide. Distance 'β' in Figure 9 is kept by vehicle j due to the discomfort caused by vehicle k, a certain proportion of which falls into the left (shoulder) lane. Three factors are expected to affect the determination of the length of distance 'β':

i) the Degree of Untidiness of vehicle k,
ii) the speed of traffic, and
iii) the gauge ratio between vehicle k and j (i.e. the ratio of the height of vehicle k to the height of vehicle j).

5.3 Issue of Slowdown

In free flow conditions, when an approaching vehicle from upstream faces untidiness on the roadway, the driver makes a decision to over(under)take by reducing/maintaining his/her desired speed or to follow the leading vehicle(s) by slowing down (Figure 10). This may lead to backward forming shockwaves which may cause a negative effect on traffic flow. The crucial point of this slowdown issue is the clarification of the decision making process of driver k. This clarification will also facilitate the three factors of Lateral Discomfort, introduced in the previous issue.

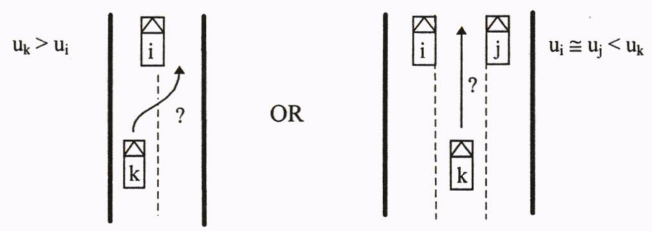


Figure 10: Demonstration of slowing down caused by untidiness.

5.4 Structure of the Simulation

If it can be proven that the combination of the effects of the accommodation and the slowdown issues produces a negative impact on traffic flow, a relationship between untidiness and throughput, validating the hypothesis shown in Figure 8, will then have been established. On the basis of these two issues, a simulation of untidy traffic flow can be performed which will basically have the framework shown in Figure 11. Excluding the details of the model and of the simulation steps, a basic algorithm of the approach can be presented as in the Appendix.

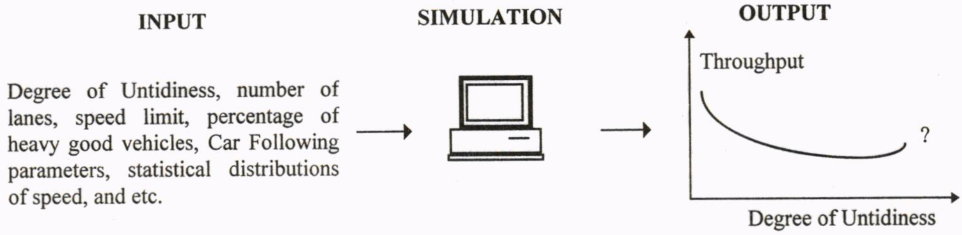


Figure 11: Framework of the simulation.

6. CONCLUDING REMARKS AND FUTURE WORK

The preceding sections discussed the problem of low lane discipline which may be a potentiality for throughput drops in untidy flow situations. To analyse the effects of untidiness, mathematical determination of the secondary Car Following distance 'β' is the crucial step of the research. After the model and simulation are successfully established, including their empirical validations which will be carried out by means of the data collected from a disorderly flowing traffic, possible applications of the research will then be implemented. For example, for multi-lane urban highways, Fisk (1990), based on some of the existing traffic flow simulation packages that take into account the lane switches, examined the question of flow allocation to competing lanes. However, untidiness has not been included in such models. By including the new dimension of untidiness, an enhancement to some of the existing simulation models, such as SIGSIM (Silcock, 1993), MULTSIM (Gipps, 1986), etc., to be applied to some road networks with relatively higher Degree of Untidiness, would be a possible employment of the research. Further clarification of Degree of Untidiness, given in Figure 6(b), can also be carried out in the future work. For instance, similar to the research by Munjal and Pipes (1971), who studied density oscillations between lanes for a given road length, longitudinal pattern of the curve of Degree of Untidiness along the highway will be investigated. Besides, regarding the relationship between flow and number of lane changings, to add a new curve, representing untidy multi-lane traffic in a developing country, to Figure 5 would be an interesting outcome of the study.

Furthermore, as Reiter (1991) revealed (Section 2.2), automation improves the quality of traffic flow in multi-lane highways. This improvement is expected to be more effective in low driving discipline motorways where unnecessarily high number of lane changings is observed. Hall (1995) stated that even under the best control scenarios, each lane change would likely result in a capacity decrement. The capacity loss for each lane change, combined with the rate at which lane changes occur, strongly influence the capacity of Automated Highway Systems. As an example, assuming a motorway on which mean trip length is 20 km, mean speed is 108 km/h and nominal capacity is 7200 vehicles/h, the effect of lane change on the capacity reduction was obtained as 20% (Hall, ibid.). Likewise, the introduction of Variable Message Signs that display "Keep in Lane" or "No Lane Changing" can especially be considered on the grounds of improved traffic safety. As Sparmann (1979) emphasised, lane changing is particularly dangerous when dense traffic is generated by bottlenecks. A strategy to facilitate these Variable Message Signs (i.e. their effect on untidiness, the distance between the two consecutive signs, their timings and etc.) will also be studied as the research proceeds.

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APPENDIX: Flow chart of the analogy