

ESTIMATION OF THE ASSIGNED TRAFFIC VOLUME IN CONSIDERATION OF SIGNALIZED INTERSECTIONS

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Abstract: The major purposes of the traffic assignment are to plan and examine the operation of road network. However, conventional static assignment methods exclude the effects of signalized intersections and deal with them as if they were grade separations. Thus, such conventional models have problems to assign traffic on routes with many intersections. We developed a traffic assignment model using the fuzzy set theory to evaluate intersectional friction as fuzzy intersection delay. The evaluation items in its estimation process are: (1) link flow, (2) approach capacity, (3) the number of oncoming (or opposing) vehicles and (4) directions at the intersection (straight through, left-turn and right-turn).

Keywords: assigned traffic volume, intersectional friction, fuzzy intersection delay.

1. INTRODUCTION

The main purposes of the traffic assignment are to estimate the traffic volume of a road network and thus to examine the plans and operation of the road network. A incremental assignment method, a handy method of the user optimizing deterministic assignment method or the user equivalent assignment method, has been used widely by road administrators because it allows easy calculation and handling. And the incremental assignment method is

generally known to provide the same assignment result as the user optimizing deterministic assignment method when the number of iteration is adequately great and no routes are with saturation flow in the network (which will be ignored in the calculation). However, in general, the conventional assignment methods represented by the incremental assignment do not deal with the effects of signalized intersections. As a result, a problem exists that a traffic volume had been assigned even to the routes that were hard to travel along (that had much intersectional friction) and that was unlikely to be used by drivers. So we thought that the incremental assignment method needs to be improved.

Some traffic assignment models that take the signalized intersections into consideration have been developed. For example, a method to express the signalized intersections with dummy nodes and dummy links (2) and a micro-method of simulation. But these assignment methods require such complicated calculations and much time to calculate when a large-scale road network is the subject, such as for inter-city traffic.

For this reason, current traffic assignments for a large-scale road network are generally being calculated regardless of the effects of signalized intersections. However, because the operation stage needs to consider the effects of these signalized intersections, the management of the road network has had difficulty in coordinating the plan and operation. From the point of view of the road administrators, a traffic assignment method that considers the effect of signalized intersections and provides enough simple calculations to calculate a large-scaled road network is required.

In our study, we modified the incremental assignment method. Because our model was developed to apply to a large scale road network to be used by the road administrator, the static method is more beneficial than the dynamic one due to its easy handling. We developed a traffic assignment model by expressing the effect of intersectional friction as the fuzzy intersection delay in the minimum path search process.

2. ESTIMATING THE INTERSECTION DELAY

2.1 Intersection Delay

Intersection delay is the time difference between real travel time and hypothetical non-stop travel time assuming no signals. Intersection delay can be explained by the queuing theory. But in reality the current traffic assignment methods disregard the effects of the intersection

delay, possibly because:

calculating for all signalized intersections in the network is not practical due to the complexity of the calculation and the time to calculate by the road administrators.

This means that the assignment calculation for a large-scale road network has had a problem in terms of efficiency.

Accordingly, in this study we used the fuzzy set theory to estimate the intersection delay because of a simple calculation that makes calculating for a large-scale road network possible, and it is comparatively reliable for its easy use.

2.2 Hypothesis to Estimate the Intersection Delay

The factors that affect the intersection delay of each direction (i.e. straight through, left-turn and right-turn) are: straight through; congestion ahead; left-turn; congestion ahead, pedestrians; and right-turn; congestion ahead, pedestrians, oncoming vehicles. The other factors that affect the intersection delay unconnected to the directions are the approach capacity and link capacity.

In our model the intersectional friction is evaluated as the intersection delay, and the intersection delay is estimated from the following four hypotheses:

- 1) Drivers recognize the intersection delay as an approximate time (fuzzy time).
- 2) The intersection delay recognized by drivers increases as the link flow connected to the intersection increases (in relation to the approach capacity).
- 3) The intersection delays of straight through and left-turn vehicles at the same intersection vary. And the two intersection delays increase in the order of straight through and left-turn as the link flows increase.
- 4) The intersection delay of right-turn vehicles is equal to that of left-turn vehicles when no oncoming vehicles. However, the more oncoming vehicles are present, the more intersection delays of right-turn vehicles occur that result in a larger intersection delay for right-turn vehicles than that of left-turn vehicles.

2.3 Process of Estimating the Fuzzy Intersection Delay

In our model we express the intersection delays by fuzzy numbers from the hypotheses given in Section 2.2. We use the following four evaluation items to estimate the intersection

delay:

- (a) Link flow
- (b) Approach capacity
- (c) Number of oncoming vehicles
- (d) Directions (straight through, left-turn and right-turn)

Straight through and left-turn vehicles: We first define the inter-link connective rate with the link flow (evaluation item (a)) and the approach capacity (evaluation item (b)). We call this inter-link connective rate the connective rate in this paper. The connective rate ($\mu(q)$) is obtained from the membership function (Figure 1). Here, q , $\mu(q)$ and C_i indicate the link flow, value of connective rate and approach capacity, respectively.

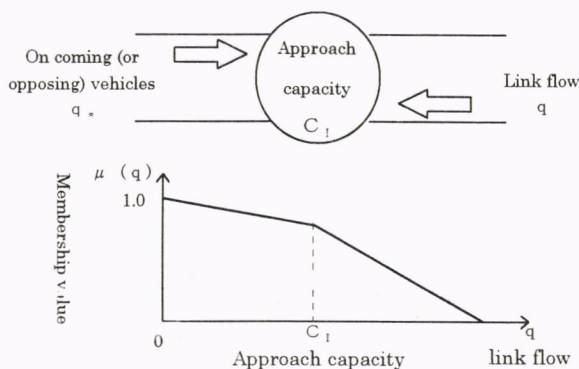


Figure 1. Inter-link connective rate at an intersection

When the link flow is 0 (when no traffic flows into the signalized intersection) the connective rate is 1; the greater the link flow, the less is the connective rate, and the connective rate approaches 0.

Right-turn vehicles: The connective rate is equal to the product of the connective rate of left-turn vehicles by oncoming vehicles (evaluation item (c)). That is, the connective rate of straight through and left-turn vehicles is $\mu(q)$ and that of right-turn vehicles is $\mu(q) \times \mu(q_s)$. Here, q_s is the number of oncoming vehicles.

This is based on the assumption that (1) the inter-link connectivity at the signalized intersection is fuzzy connective, (2) the connectivity rate of straight through and left-turn vehicles depends on the link flow and approach capacity, (3) the connective rate of right-turn vehicles depends on the link flow, approach capacity and the number of oncoming vehicles.

Next, the intersection delays, which are the basis of the estimation, are expressed by triangular fuzzy numbers at all signalized intersections (Figure 2).

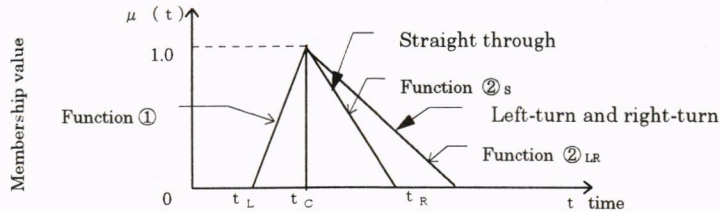


Figure 2. Basis to estimate the intersection delay

Here, t_c is the mean of the triangular fuzzy numbers, and t_R and t_L are the right and left spreads of the triangular fuzzy numbers, respectively. The right spread is expressed by Equation (1). Here, β expresses the size of the right spread. We use β_s for straight through and β_L for both left- and right-turn vehicles ($1 \leq \beta_s < \beta_L$).

$$t_R = \beta \cdot t_c \quad \beta \geq 1 \quad (1)$$

The left spread is expressed by Equation (2) using parameter γ , which expresses the size of the left spread (3 and 4).

$$t_L = \gamma \cdot t_c \quad 0 < \gamma < 1 \quad (2)$$

The fuzzy intersection delay, which is the basis of the estimation (Figure 2), is a function on the left side of the mean t_c , ①, and is turned and inverted at the axis of $\mu(t) = 0.5$ to define a new function ①'. Functions on the right side of the mean t_c , ②_s and ②_{LR}, (Figure 2) shift to the right by the same distance between t_L and t_c , which is the time between the left spread and the mean (functions ②'_s and ②'_{LR}). New functions made in this way (Figure 3) are set as standard functions to estimate the intersection delay.

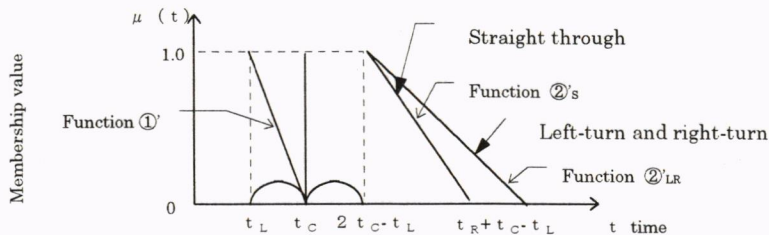


Figure 3. Standard function to estimate the intersection delay

In our model, the link travel time obtained from a B.P.R. (Bureau of Public Roads) function is used. And the route travel time is the sum of the fuzzy intersection delays and link travel times. The method to calculate the route travel time as the total time recognized by drivers and the time obtained from the B.P.R. function may not be the proper way, but we think if the recognition mechanism of drivers for time is clarified, this method will go a long way to being proper. In the process of a minimum path search, three minimum path candidates are selected and the smallest one is selected as the minimum path. Because a signalized intersection is not expressed by dummy nodes and dummy links, a sub-minimum path of an optional minimum path does not always become a minimum path.

3. CALCULATION EXAMPLE OF THE ROUTE TRAVEL TIME

This section describes the method to calculate the travel time for a route, using the network in Figure 5 as an example. Here, $N(t_L, t_C, t_R)$ is a fuzzy number and t_L, t_C, t_R are the left spread, mean and right spread of the fuzzy number, respectively. T_{ij} is the required link time from Node i to Node j and is calculated by using the BPR function.

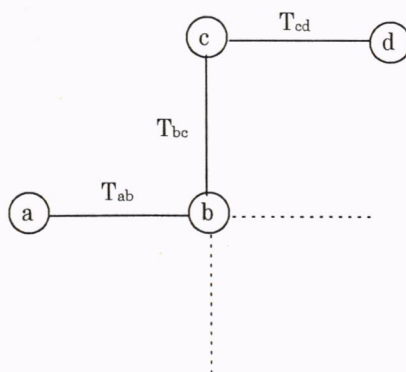


Figure 5. Network

The travel time required for the route $a \rightarrow b$.

$$T_{a \rightarrow b} = T_{ab} \quad (3)$$

The travel time required for the route $a \rightarrow b \rightarrow c$ (left-turn at intersection b).

$$\begin{aligned} T_{a \rightarrow b \rightarrow c} &= T_{ab} + N(t_L^b, t_C^b, t_R^b) + T_{bc} \\ &= N(T_{ab} + t_L^b + T_{bc}, T_{ab} + t_C^b + T_{bc}, T_{ab} + t_R^b + T_{bc}) \end{aligned} \quad (4)$$

where, $N(t_L^b, t_C^b, t_R^b)$ is the fuzzy intersection delay when a left-turn is made at intersection b.

The travel time required for the route $a \rightarrow b \rightarrow c \rightarrow d$ (left-turn at intersection b, right-turn at intersection c).

$$\begin{aligned} T_{a \rightarrow b \rightarrow c \rightarrow d} &= T_{a \rightarrow b} + N(t_L^c, t_C^c, t_R^c) + T_{cd} \\ &= N(T_{ab} + t_L^b + T_{bc} + t_C^c + T_{cd}, T_{ab} + t_C^b + T_{bc} + t_C^c + T_{cd}, T_{ab} + t_R^b + T_{bc} + t_R^c + T_{cd}) \end{aligned} \quad (5)$$

where, $N(t_L^c, t_C^c, t_R^c)$ is the fuzzy intersection delay when a right-turn is made at intersection c.

The fuzzy intersection delay used in this example is the value obtained by the estimation process in 2.3. The travel time required for the route becomes the fuzzy number after crossing the intersection. In addition, performing the minimum path search is possible by using the center of gravity as the representative value for the travel time for the fuzzy route.

4. SETTING PARAMETERS FOR THE FUZZY INTERSECTION DELAY

Our model requires to set parameters, including the mean, left spread and right spread of the fuzzy intersection delay and the membership function of the connectivity rate. They need to be set appropriately to calculate the proper traffic assignment. In this section, an example method of setting these parameters are referred to.

The mean of the fuzzy intersection delay (t_c) is the average of the intersection delay when the link flow is 0 (when the connective rate is 1). So the mean of the fuzzy intersection delay is set by Equation (6).

$$t_c = \frac{(c - g)^2}{2c} \quad (6)$$

where, c and g are the cycle time and the effective green time in the phase of the signalized intersection, respectively (5).

In the estimation process of the intersection delay described in the Section 2.2, the intersection delay when the connectivity rate is 1 is a fuzzy intersection delay symmetrical to the mean. The spread or the recognition spread of this fuzzy intersection delay is determined by parameter γ . However, we do not have the data of the recognition spread

time. Here, we assume that drivers recognize time with a 30 percent recognition spread time, and set γ as 0.7.

On the other hand, the center of gravity as a representative value of the fuzzy intersection delay (t_G) is expressed by Equation (7).

$$t_G = -\frac{t_c(\beta - \gamma)}{3}\mu + \frac{t_c(\beta - \gamma + 3)}{3} \quad (7)$$

where, μ is the connective rate. Equation (7) indicates that the center of gravity of the fuzzy intersection delay is a linear function of the connective rate μ . Here the membership function of the connectivity rate is set at 1 when the link flow is 0; set at k ($0 < k < 1.0$) when the link flow is equal to the approach capacity (C_1); and set at 0 when the link flow is αC_1 ($\alpha > 1$) (Figure 6).

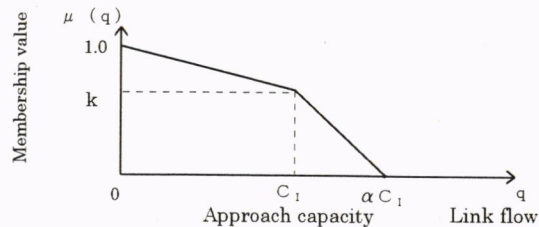


Figure 6. Membership function of connective rate

In this case, the center of gravity of fuzzy intersection delay (t_G) is expressed by Equations (8) and (9).

when $0 \leq \frac{q}{C_1} \leq 1.0$

$$t_G = \frac{t_c(\beta - \gamma)}{3} \frac{q}{C_1} + t_c \quad (8)$$

and when $1.0 < \frac{q}{C_1} \leq \alpha$

$$t_G = \frac{t_c(\beta - \gamma)k}{3(\alpha - 1)} \frac{q}{C_1} + \frac{t_c(\beta - \gamma + 6)k}{3} - \frac{t_c(\beta - \gamma)k}{3(\alpha - 1)} \quad (9)$$

According to the Equations (8) and (9), the center of gravity of the fuzzy intersection delay is a linear function of the ratio of q/C_1 (link flow to approach capacity) (Figure 7).

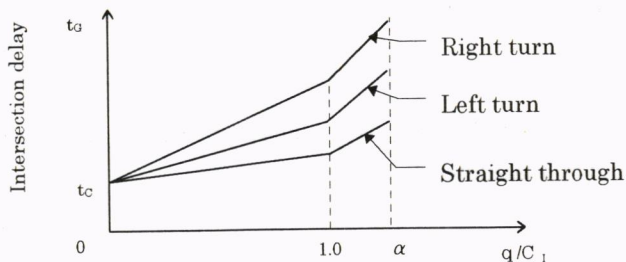


Figure 7. Intersection delay varied from directions

Because, the gradients of these functions depend only on the parameters β (straight through: β_s , left-turn: β_L , right-turn: β_R). The gradients of Equation (8) (the gradient at the range of $0 \leq q/C_1 \leq 1.0$ for these functions in Figure 7) are set using the linear approximation function of the average intersection delay function. The average intersection delay function is expressed by Equation (10).

$$t_d = \frac{c(1 - g/c)^2}{2(1 - (g/c) \cdot (q/C_1))} \quad (10)$$

$$0 \leq \frac{q}{C_1} \leq \alpha$$

where, t_d is the average intersection delay. Figure 8 shows the average intersection delay function and its linear approximation function.

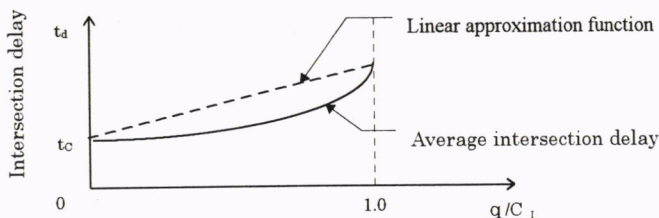


Figure 8. Average intersection delay function and its linear approximation function

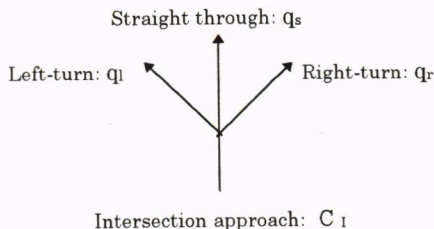


Figure 9. Design approach capacity

The average intersection delay function is determined using the design capacity of a signalized intersection (C_I) (Figure 9). The gradient of its linear approximation function is determined from Figure 8. Here, we express the gradient by m . We assume that the gradient, m , is a weighted average of the ratio of the directions capacity to the intersection approach capacity (straight through: q_s/C_I , left-turn: q_l/C_I and right-turn: q_r/C_I). Here, the gradients of the intersection delay for straight through, left-turn and right-turn are m_s , m_l and m_r , respectively.

$$m = \frac{q_s}{C_I} m_s + \frac{q_l}{C_I} m_l + \frac{q_r}{C_I} m_r \quad (11)$$

From Figure 9,

$$q_s + q_l + q_r = C_I \quad (12)$$

Assuming that the ratios among the gradient varied from the directions are equal to the ratios among the inverse number of the capacities varied from the directions (Equation (13)), then

$$m_s : m_l : m_r = \frac{1}{q_s} : \frac{1}{q_l} : \frac{1}{q_r} \quad (13)$$

The gradients m_s , m_l , m_r are obtained from Equations (11), (12) and (13). Next, the parameters straight through: β_s , left-turn: β_l , and right-turn: β_r are obtained by making the gradients of Equation (8) equal to the gradient m_s , m_l and m_r , respectively. The parameter β_r (right-turn) is not a parameter in our assignment model, but it is required in setting parameters.

In our model, when the traffic volume exceeding the approach capacity ($C_I \leq q \leq \alpha C_I$) flows into a signalized intersection, the intersection delay is defined. In the Equation of Webster (5), generally used for estimating the intersection delay when the traffic volume exceeding the approach capacity ($C_I \leq q \leq \alpha C_I$) flows into an intersection, the intersection delay is infinite, because in the Equation of Webster, an overflow is supposed to continue forever. However, in our model, we supposed that an overflow never continues forever and because of that the intersection delay occurs when the traffic volume q is $C_I \leq q \leq \alpha C_I$.

This is based on the idea that the intersection delay when the traffic volume exceeding the approach capacity for ($C_I \leq q \leq \alpha C_I$) flows into an intersection depends on the duration of overflow (6). In our model, the connectivity rate (k) when the traffic volume equals the

approach capacity determines the intersection delay for ($C_i \leq q \leq \alpha C_i$). That is, when the connective rate k is larger (close to 1), the intersection delay is larger. Conversely, when the connective rate k is smaller (close to 0), the intersection delay is smaller, because the gradients of Equation (9) depend on the connective rate k when the traffic volume is equal to the approach capacity. Therefore, the connective rate k can be set according to the traffic conditions (or signalized intersection conditions).

5. EXPRESSION OF INTERSECTIONS BY MATRICES

In the Dijkstra method (7), the columns and rows of a two-dimensional matrix are ordinarily represented by the node numbers, and the evaluated value of the link (or travel time) is assigned to matrix elements to perform computations for the minimum path search.

On the other hand, in this model the intersection delay differs according to the directions (straight through, left-turn, right-turn) at the intersection. As a result, clarifying the directions at the intersection during the process of a minimum path search is necessary. In addition to the conventional two-dimensional matrix (the evaluated value is assigned), the node-node connection relationship, straight through relationship, left-turn and right-turn relationships of the network are expressed here as a matrix. Figure 10 is an example of the network. These relationships are described hereafter as a node-node connection matrix (Figure 11), straight through matrix (Figure 12), left-turn matrix (Figure 13) and right-turn matrix (Figure 14), respectively.

The definition of a node-node connection matrix is a matrix that expresses the connections between the nodes. And the definition of straight through, left-turn and right-turn connection matrices are matrices that express the directions (straight through, left-turn and right-turn) between three nodes sequentially connected corresponding to the node-node connection matrix.

In the node-node connection matrix, the node numbers of the network are assigned to a row, and node numbers connected to the row node numbers are assigned to columns of the row.

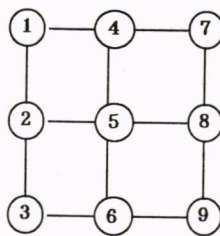


Figure 10. Network

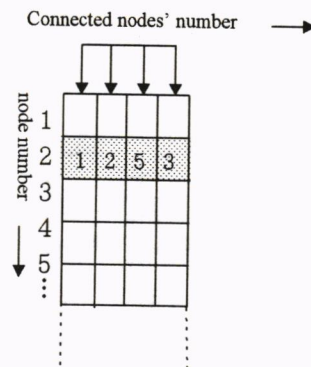


Figure 11. Node-node connection matrix

Figure 11 also illustrates the node-node connection for node 2. This demonstrates that nodes 1, 2, 5 and 3 are connected from node 2. Suppose that the traffic advances from node 2 to node 5 in the network shown in Figure 9. At this time, since node number 5 is assigned to row 2 column 3 of the node-node connection matrix, node number 8 is assigned to row 2 column 3 of the straight through matrix ($2 \rightarrow 5 \rightarrow 8$: straight through (Figure 12)). In the same way, node numbers 4 and 6 are assigned to row 2 column 3, respectively, of the left turn matrix and right turn matrix. ($2 \rightarrow 5 \rightarrow 4$: left turn (Figure 13), $2 \rightarrow 5 \rightarrow 6$: right turn (Figure 14)).

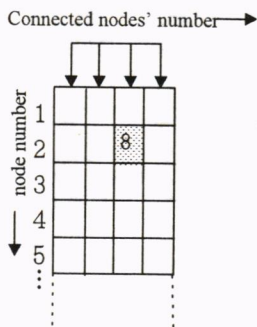


Figure 12. Straight through matrix

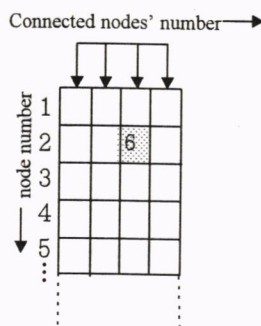


Figure 13. Left turn-matrix

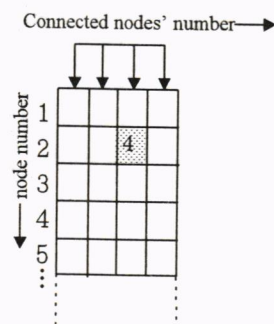


Figure 14. Right-turn matrix

The directions (straight through, left turn and right turn) at the intersection can be expressed in this manner.

6. CALCULATION OF ASSIGNED VOLUME AND CONSIDERATION

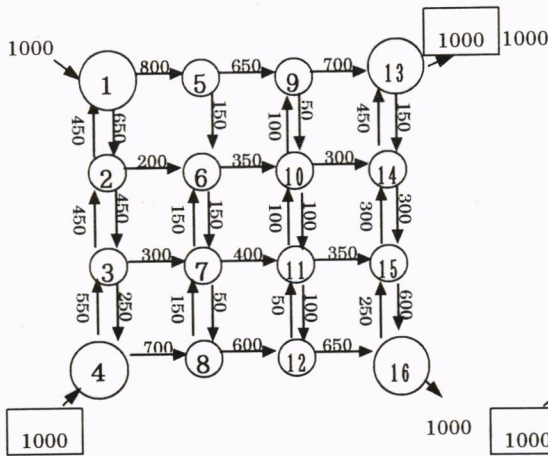


Figure 15. Assignment result of our model
(Number of iteration=20)

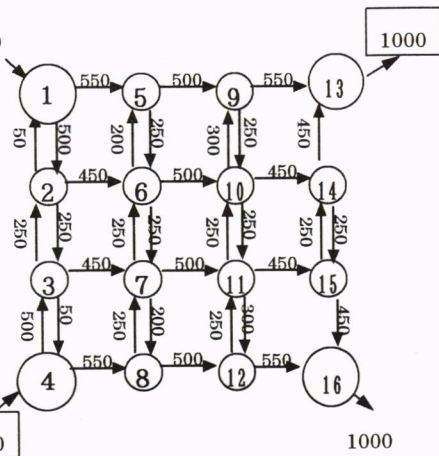


Figure 16. Assignment result of
incremental assignment method
(Number of iteration=20)

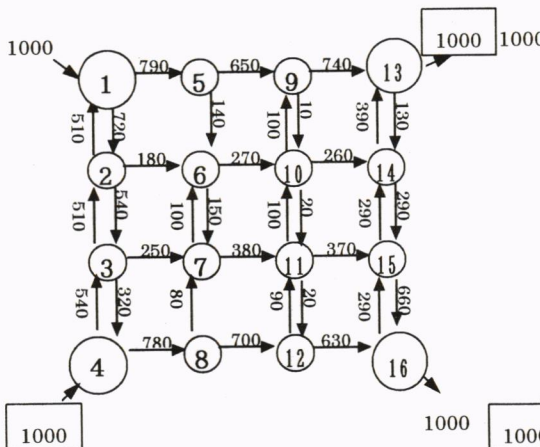


Figure 17. Assignment result of our model
(Number of iteration=100)

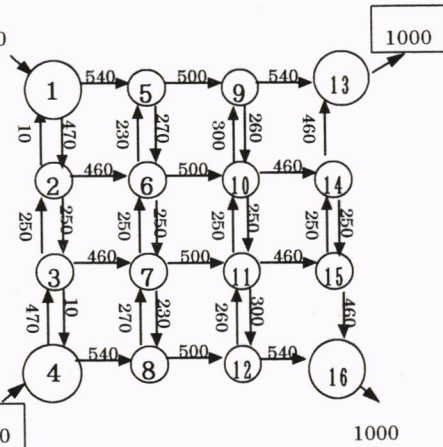


Figure 18. Assignment result of
incremental assignment method
(Number of iteration=100)

The Figures 15 and 16 show the assignment result of our model and the incremental assignment method ordinarily used in the calculation to examine the validity of our model. A subjective network is a virtual network consisting of 16 nodes and 24 links. The free travel time of each link is set equally at 40 seconds. The capacity of each link is set equally at 1600 vehicles per hour, and the capacity of each intersection is set equally at 800 vehicles per hour. Two pairs of O.D. (Origin and Destination) traffic volumes (1000 vehicles each),

which origins are at Nodes 1 and 4, and their destinations are Nodes 16 and 13 assigned to the network to reflect the effect of signalized intersections. The number of iteration is set at 20 and the traffic is equally assigned (one repeat of traffic volume is 50 vehicles).

The phase of signalized intersection is set at the cycle time of 60 seconds and the effective green time is 30 seconds. And the parameters for fuzzy intersection delay obtained by setting the design traffic capacity at 800 vehicles per hour (straight through: 500 vehicles per hour, left-turn: 200 vehicles per hour, and right-turn: 100 vehicles per hour) are used. The membership function of the connective rate is set at 1 when the link flow is equal to 0, and 0.5 when the link flow is equal to the approach capacity (C_i), and 0 when the link flow is 1200 ($\alpha=1.2$).

On the other hand, Figures 17 and 18 show the assignment results of our model and of the incremental assignment method calculated to show more strict assignment results when the number of iteration is 100 (one iteration of traffic volume is 50 vehicles). Other conditions are the same as for Figures 15 and 16.

Calculation results:

- 1) In our model, the link flows from Node 2 to Node 6 and from Node 3 to Node 7 are less than that of the incremental assignment method, because our model includes the effect of oncoming vehicles and consequently the right-turn traffic traveling from Node 4 to Node 1 decreases.
- 2) Also, in our model, the link flow of the outside link of the network is more than that of the incremental assignment method, because the intersection delay of straight through vehicles is less than that of left-turn or right-turn vehicles, so the routes consisting of as many straight through routes as possible tend to be selected.
- 3) In our model, the influence of number of iteration on the assignment result was more than the incremental assignment method. As the number of iteration decrease, the traffic volume assigned one time increases, our model is more sensitive to the differences in traffic volume assigned at one time than the incremental assignment method, because our model includes not only the link travel time, but also the intersection delay.
- 4) In the Figures of 16 and 18, the four routes: from Node 1 to Node 13 ($\textcircled{1} \rightarrow \textcircled{5} \rightarrow \textcircled{9} \rightarrow \textcircled{13}$), Node 1 to Node 4 ($\textcircled{1} \rightarrow \textcircled{2} \rightarrow \textcircled{3} \rightarrow \textcircled{4}$), Node 4 to Node 1 ($\textcircled{4} \rightarrow \textcircled{3} \rightarrow \textcircled{2} \rightarrow \textcircled{1}$), and from Node 4 to Node 16 ($\textcircled{4} \rightarrow \textcircled{8} \rightarrow \textcircled{12} \rightarrow \textcircled{16}$) are congested because of two pairs of O.D. The vehicles flowing into the network tend to avoid congestion. The link flows from Node 6 to Node 5 and Node 7 to 8 become smaller (or nonexistent) because the two

routes from Node 5 to Node 13 ($\textcircled{5} \rightarrow \textcircled{9} \rightarrow \textcircled{13}$) and from Node 8 to Node 16 ($\textcircled{8} \rightarrow \textcircled{12} \rightarrow \textcircled{16}$) are congested. The vehicles flowing into the network tend to exit at Nodes 9 or 13 from Node 10, or at Nodes 12 or 15 from Node 12. The traffic volume with an O.D. pair having Node 1 as the origin and Node 16 as the destination tends to flow toward Node 2, because the route from Node 1 to Node 13 is congested (because it is also used by traffic volume with an O.D. pair having Node 4 as the origin and Node 13 as the destination). The greater the number of iteration, the greater is this tendency (720 out of 1000 vehicles flow into Node 2 in Figure 16). Consequently, the greater the number of iteration, the fewer the link flows from Node 11 to Node 12 (Figure 16).

- 5) The right-turn conflict with the oncoming vehicles when such traffic exists and the tendency of drivers to select a route with fewer turning (left-turns and right-turns) are expressed in our model, but which are not considered in the incremental assignment method. Thus our model expresses a more realistic traffic situation than the incremental assignment method.

7. CONCLUSIONS

In this study, we developed a traffic assignment model to include consideration of intersection delays varying from the different directions (straight through, left-turn and right-turn). In our model, we use the Dijkstra method as a minimum path search algorithm to reduce the calculation load. Because of that, our model is able to be used to calculate the assigned volume in large scale networks such as inter-city traffic and is easy to handle for road administrators to manage and operate a road network.

Further study is necessary to calculate the assigned volume in a larger scale network. This model uses the methods of the fuzzy set theory during the process of the minimum path search. In this method, a traffic volume is assigned to the route with the smallest representative value of the fuzzy route travel time on an "all or nothing" basis. If the actual traffic conditions are considered, some drivers may select not only the center of gravity of the fuzzy route travel time, but also the shortest route of the right and left spreads. For this reason, in addition to the intersection delay the assignment needs to use the fuzzy set theory in our further study.

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