AN INTEGRATED DYNAMIC TRAFFIC ASSIGNMENT MODEL FOR RESPONSIVE SIGNAL CONTROL AND VARIABLE MESSAGE SIGN

Yongtaek Lim
Research Associate
Department of Urban Transportation
Seoul Development Institute
San4-5, Yejang-dong, Chung-ku
Seoul 100-250, Korea
Fax:+82-02-726-1291
E-mail: limyt@plaza.snu.ac.kr

Seungjae Lee
Professor
Department of Urban Engineering
Seoul City University
Jeonnong-dong, Dongdaemum-ku
Seoul, Korea
Fax:+82-02-215-5097
E-mail: sjlee@scucc.scu.ac.kr

abstract : An integrated dynamic traffic assignment model and its solution algorithm are developed. Responsive signal control policy and VMS strategy which are adaptive to traffic conditions are also included in the model. The model can be formulated and solved as a tri-level program. The developed integrated model will be expected to be useful to develop and evaluate the diverse strategies of Intelligent Transport Systems.

1. INTRODUCTION

In order to avoid or reduce the urban traffic congestion, diverse traffic management schemes are considered. Some of them are making best use of available road capacity, signal control and provision of information. The usage of real-time traffic information and Advanced Traffic Management Systems within the framework of Intelligent Transportation System, in particular, has become a powerful tool as a possible solution to the ever growing congestion problem. Dynamic traffic assignment model is required to evaluate the impact of various traffic management strategies and will play an important role in the development of real-time traffic control systems and providing information systems.

The effect of intersection on travel cost is of primary importance in urban networks because most of the travel time be spent at the intersection, but conventional equilibrium route choice is not fully considered this fact. Since Allsop(1974) suggested that the effects of signal settings on the traffic flow pattern should be taken into account by combining traffic control and route choice, many researchers have developed this combined model.

Recently various approaches have been developed to the traffic information systems within the framework of ITS. In terms of providing information, the approaches are classified into individual in-vehicle route guidance system and collective system such as VMS(Variable Message Sign). In the collective system, drivers may receive information from Variable Message Sign distributed at a key location in the network. In contrast to the in-vehicle systems, the message content will have general value which individual drivers must interpret. The subsequent driver's decision will vary with their level of knowledge of the network and their ability so as to determine the implications of the message for their own journey. The message enables to provide driver's useful information such as the network condition and parking guidance. One of the principal issues with information is what kind of information is provided. Several works showed that the effects of traffic informations can be varied with information provision strategy.

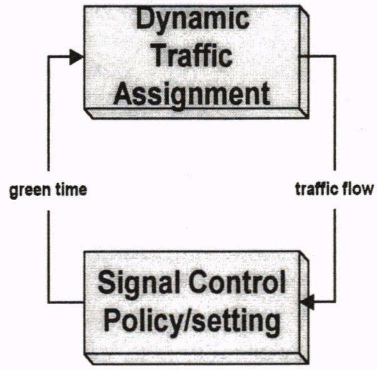
The objective of this paper is to develop an integrated dynamic traffic assignment model for responsive signal control and variable message sign information. Responsive signal control and VMS information strategy are adaptive in this paper due to traffic conditions. A mathematical formulation and its solution algorithm are also proposed for the model. This paper also show that developed solution algorithm converge to Wardrop equilibrium. The model will be expected to be a useful tool for the assessment of dynamic traffic management strategies. Next sections show the structure of an integrated model and illustrate the interaction among subsystems.

2. STRUCTURE OF THE COMBINED DYNAMIC TRAFFIC ASSIGNMENT MODEL

2.1 Combined signal control and traffic assignment model

The delay at intersection takes an important part in link travel cost of traffic assignment because most of the travel cost is intersection delay. There are two main approaches to the solution of the combined signal control and traffic assignment problem: the global optimization method and iterative optimization method. The global optimization model seeks signal control variable such as green time, within the equilibrium condition. Gartner et al(1980), Fisk(1984), Marcotte (1983) and others proposed solution algorithms on small networks. Recently Yang et.al(1994) also proposed a bi-level programming for somewhat different combined model with traffic assignment and ramp metering. The main difficulty with the global optimization methods is that, firstly, they are restricted to the small size of the network problem. Secondly, they can represent only approximate unrealistic modelling because they need strong assumptions of a cost function and a network structure for better behaviour. Lastly, their solutions guarantee only local solutions because the problem has a non-convex constraint and a non-linear objective function.

The iterative approach is to solve signal control and traffic assignment alternately. The signal control problem yields some decision variables such as green time and cycle time under fixed flows, and then the traffic assignment problem solves the user behaviour condition to give traffic flows when some signal variables are fixed. This iterative procedure is repeated until mutually consistent values of the variable are obtained(see Figure 1). Allsop et al(1977) introduced this iterative computational procedure for the combined signal setting and road traffic assignment problem. Smith(1981) proposed a signal control policy that guarantees the existence of a traffic equilibrium that is consistent with it. Smith et al(1987) carried out a comparative stability test between Smith's policy and others. Cantarella et al(1991) also proposed an iterative procedure in which traffic signal setting is calculated in two successive steps: green timing and scheduling at each junction and signal co-ordination on the network.



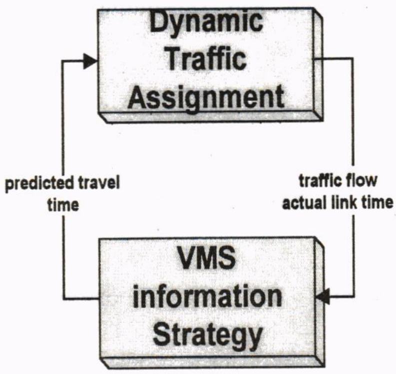
< Figure 1 > Iterative optimization approach for signal control and traffic assignment

2.2 Combined information and traffic assignment model

Before formulating a combined traffic assignment model with information, there are some important factors we consider in traffic information as follows. Driver information systems can vary in the nature of the information they provide. That is, they can either provide status or guidance information. In addition, the information they provide can be either historical, current or predictive. The exact nature of the information will influence how people will respond over both the in real-time and over the long run.

Another factor is the driver's compliance of the information. Information systems can have a significant impact on both pre-trip and en-route adjustment behavior. However, an information system cannot be used to achieve any desired response pattern. The most important reason for this is that traveler compliance to information directives cannot be guaranteed. In particular, it can be argued that an information system can only be effective if the information being provided is perceived as being both reliable, in the sense that what is predicted occurs, and useful. Otherwise, its use may be discontinued, rendering it ineffective.

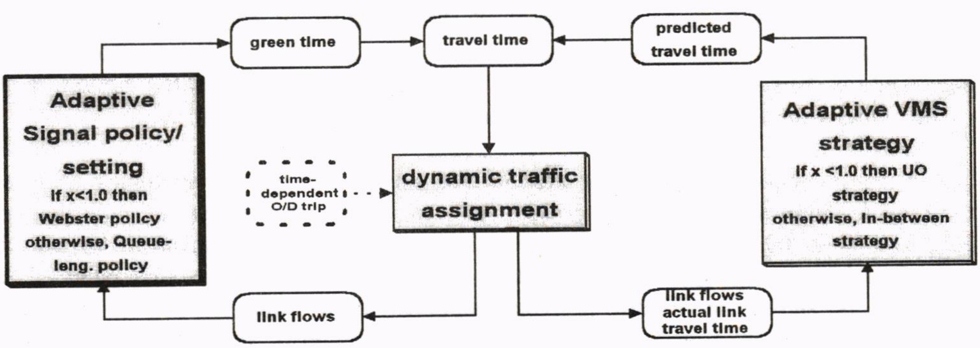
The main researches in the paper are focused on the developing strategies of how to provide information to the drivers and to assess the effect of information provision with the compliance of driver. Just as the iterative approach for signal control and traffic assignment, an iterative approach is developed in the paper. The approach is shown in Figure 2. This approach is a kind of bi-level problem consisting of dynamic assignment problem as upper level and VMS information strategy as lower level. These two problems are solved interactively until mutually consistent solutions are reached.



< Figure 2 > Iterative approach for VMS strategy and traffic assignment

3. INTEGRATED DYNAMIC MODEL

The developed model in this paper consists of three modules such as responsive signal control, variable message sign and dynamic traffic assignment as shown in Figure 3 below. Responsive signal control is used to determine green time in order to minimise the total delay of the system. Variable message sign is used to provide short-term route following information in order to avoid traffic congestions ahead of some alternative routes. These signal control and information provisions are adaptive to traffic conditions on road networks. If the degree of saturation x is below 1.0, signal control policy chooses Webster policy. Otherwise, selects Queue-length policy. VMS informations also have the same strategy as signal control policy. They are, however, under the conditions of driver's dynamical behavioral adjustment mechanism. All of the three-level routines calculate interactively until mutually consistent traffic flows, green time are obtained.



< Figure 3 > Systematic Diagram for Integrated model

3.1 Dynamic traffic assignment model

Real-time dynamic route choice model is developed in order to reflect the driver's time-varying route choice behaviour according to the signal control and VMS information. Driver behaviors are varied with the travel cost which is combined actual link travel cost (running cost and junction delay) and short-term predicted travel cost. The travel costs are updated every short time period, for example 15 minute intervals, and the model simulates the network traffic conditions.
The link travel time is as follows:

$$T_a(f^t,\lambda^t)=(1-\delta)A_a(f^t,\lambda^t)+\delta S_a(f^{t'},\lambda^{t'}) \tag{1}$$

Where $T_a(f^t,\lambda^t)$ is the total travel cost on link a at time t, comprising of actual travel cost $A_a(f^t,\lambda^t)$ and short-term predicted cost, $S_a(f^{t'},\lambda^{t'})$. δ is a parameter reflecting driver's behaviour. In the case of that the value of δ is 0, then drivers follow only actual travel cost that they have taken previous days. If δ is 1, vice versa. $A_a(f^t,\lambda^t)$ and $S_a(f^{t'},\lambda^{t'})$ are the function of traffic flow f and green time split λ as follows respectively.

$$A_a(f^t,\lambda^t)=t_0+\alpha[c_a(f_a^t)-t_o]+(1-\alpha)d_a(f^t,\lambda^t) \tag{2}$$

$$S_a(f^{t'},\lambda^{t'})=\beta_1A_a(f^t,\lambda^t)+\beta_2A_a(f^{t-1},\lambda^{t-1})+\beta_3A_a(f^{t-2},\lambda^{t-2}) \tag{3}$$

where, $c_a(f_a^t)$ is BPR cost function and $d_a(f^t,\lambda^t)$ is expanded Webster delay function.

3.2 Adaptive Signal control policy/setting

Since 1950s, various signal control policies have been proposed. Some of them are Webster's equisaturation policy(1958), Allsop's delay minimization (1971), Smith's capacity maximization(1979) among others. Most popular signal policy in traffic engineering is Webster' policy which is setting the green split to each approach in proportion to the traffic volumes. The policy is based on the premise that overall delays at a junction are approximately minimal when the v/c ratio of each approach is equal, which is known as equisaturation policy. These signal policies, however, are applicable only under-saturation flow rate at intersection. It is known at over-saturated state that minimizing queue-length or avoiding blocking upstream intersection are more important. A Queue-length signal policy for reducing the queue-length is developed. The signal policy determine the green time according to the queue-lengths of relating approaches. The assessment of the Queue-length signal policy and comparison with others are elsewhere(Lim,1997).

Webster signal policy at non-saturated state and Queue-length signal policy at over-saturated state are included to the model in the paper. Expanded Webster delay function, which is extended at the degree of saturation x^* in order to reflect the oversaturated state, is applied. Webster policy and Queue-length policy are formulated in the case of 4-phases as follows. $Eq_i.$ stands here for " equate for all approach i ".

Webster signal policy:

$$Eq_i. \quad \frac{f_i}{\lambda_i s_i} \qquad i=1,..,4 \tag{4}$$

Queue-length signal policy:

$$Eq_i. \quad s_i d_{oi} l_i \qquad i=1,...4 \tag{5}$$

where f_i, λ_i, s_i are the traffic flow, green time split and saturation flow rate on approach i respectively. d_{oi} is the expanded second term, or random term, of Webster delay function and l_i is link length on approach i.

3.3 Adaptive traffic information provision strategy

Traffic information plays a key role in drivers' route choice behaviors and it is classified into individual system and collective system. On the other aspect, the provision of traffic information is also to fall into three strategies from the viewpoint of its objectives; The general objective is almost always to minimize individual's travel time(User Optimality). Another objective is to minimize the total travel time of the network system as a whole(System Optimality) and the other is In-between UO and SO. Lim(1997) showed that the effects of traffic information could be varied according to what kind of information provide to drivers. He concluded that In-between traffic information is more effective than others at higher congested conditions and careful consideration should be given for determining information provision strategy.

The paper selects VMS as a collective information system and provide the short-term predicted travel time information to drivers. For accounting of changing traffic conditions, this paper also introduce the adaptive traffic information provision strategy to the model such as UO(user optimality) strategy at non-saturated traffic condition and In-between strategy at saturated condition. UO strategy use the *mean travel time* as a link cost function and In-between strategy adopt $\gamma *$ *marginal travel time* as a link cost. Where γ is a parameter($0.0<\gamma<1.0$). Sensitivity of drivers' following to the routing information can be tested, as the drivers' compliant parameter δ increases incrementally. The short-term predicted travel cost is formulated as a linear polynomial equation:

$$S_a(f^{t'}, \lambda^{t'}) = \beta_1 A_a(f^t, \lambda^t) + \beta_2 A_a(f^{t-1}, \lambda^{t-1}) + \beta_3 A_a(f^{t-2}, \lambda^{t-2}) \tag{6}$$

$$\sum_{i=1}^{n} \beta_i = 1 \,, \qquad i=1 \,, \,.\,.\,.\,.\,.\,, n \tag{7}$$

Where $S_a(f^{t'}, \lambda^{t'})$ is short-term predicted travel time on link a at time t, which is based on weighted averages of the travel times on current and previous times.

4. FORMULATION AND SOLUTION ALGORITHM

In the case of that signal delay is included into traffic assignment model, the link travel function becomes non-separable and asymmetric because the link cost is a function of the other link flows. Smith(1979) showed that Wardrop's condition of user equilibrium is equivalent to a variational inequality formulation. The model in this paper was formulated in the form of variational inequality as follows:

$$T(f^{t*},\lambda^t)\cdot(f^t-f^{t*})\geq 0 \qquad (8)$$
$$f^{t*}\in S\cap D,\ f^t\in D,\ \lambda^t\in S$$

Where f^{t*} is a flow at equilibrium state. $T_a^t(f^{t*},\lambda^t)$ is link travel cost on link a at time t. λ^t is green split at time t. S and D are convex feasible supply and demand sets respectively. The formulation in equation(8) was verified elsewhere by author(Lim,1997).

4.1 Solution Algorithm

Several algorithms have been proposed for solving non-separable traffic assignment problem. Among all methods currently available, a diagonalised algorithm is by far the most widely used. The diagonalised algorithm gives a separable cost function that has a diagonal Jacobian matrix of the full cost function. In the diagonalised algorithm, the influence of link interaction is ignored within each iteration, and the separable cost functions are updated in this respect at the end of each iteration.

For solving the dynamic traffic assignment model, heuristic network loading method proposed by Janson(1991) is introduced in the solution algorithm. Developed solution algorithm in this paper is different from that of Janson in that the algorithm converge to Wardrop equilibrium state.

The following is a heuristic diagonalised algorithm developed in the paper.

[step 0] Initialization
iteration n=0,
time t=1
feasible point $f^{t,n}\in D$,
[step 1] Subproblem(diagonalization)
1-1. check the degree of saturation x
if x < 1.0 then calculate green split($\lambda^{t,n}$) based on Webster signal policy and
calculate short-term predicted travel time $S_a(f^{t',n},\lambda^{t',n})$ based on UO strategy
otherwise, calculate green split based on Queue-length policy and
calculate short-term predicted travel time based on In-between

strategy

1-2. calculate link travel time

$$T_a(f^{t,n}, \lambda^{t,n}) = (1-\delta)A_a(f^{t,n}, \lambda^{t,n}) + \delta S_a(f^{t',n}, \lambda^{t',n})$$

1-3. network loading(heuristic method)

(1) calculate projected link volume

$$y_a^{t+m} = \theta^t \; w_{t-1}^{t+m} f_a^{t-1} + (1-\theta^t) \; w_t^{t+m} f_a^t \;, \quad a \in A,\; m \geq 0, t+m \in T$$

(2) searching the minimum path based on projected link flow(y_a^{t+m})

(3) perform all-or-nothing assignment

1-4. convergence check

if $G(f^{t,n}, \lambda^{t,n}) = T(f^{t,n}, \lambda^{t,n}) \cdot (f^{t,n} - u^{t,n}) \approx 0$, stop : optimum solution is f^{tn}
otherwise, goto step 1-1.

[step 2] Master problem
if $f^{t,n+1} \simeq f^{t,n}$, stop
otherwise, n=n+1 and goto [step 1]

[step 3]
if t $\geq$ time periods , stop
otherwise, t=t+1 and goto [step 1]

4.2 Convergence Test

A numerical example is presented to test the convergence of the model. The example network shown in Figure 4, includes one VMS and one signalized intersection. The input data such as link capacity, free-flow cost and signal settings are shown in Table 1. It is assumed that there is one origin-destination pair from node 1 to node 4.

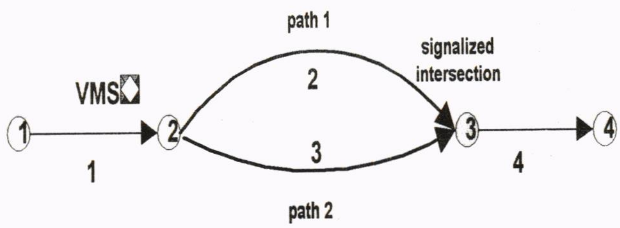


Figure 4. Example Network

Table 1. Input data

● network data

link number	1	2	3	4
free-flow cost	60	60	60	60
capacity	2000	1000	1000	2000
link length	200	500	300	200

● signal data

cycle length	60 sec(2-phase)
lost time	8 sec
eff. green	52 sec
min. green	7 sec

● others

$\beta_1 = 0.5$ $\beta_2 = 0.3$ $\beta_3 = 0.2$

trip demand = 300veh/hr.

It is proven that if the value of gap function in variational inequality get to zero value, the solution algorithm reach Wardrop equilibrium(Hearn, 1982). Figure 5 show the transition of gap value with varying trip demands. As shown in figure, gap value settle down to zero after 15 iterations on the whole. It is, therefore, shown that the solution algorithm, heuristic diagonalized algorithm converge to Wardrop equilibrium stably.

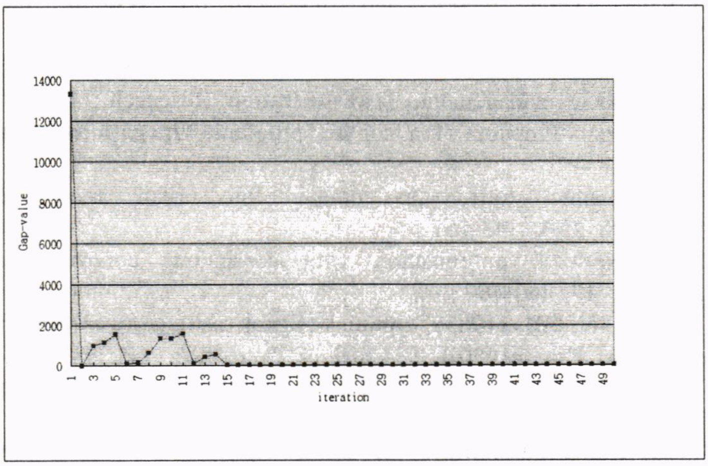


Figure 5. evolution of Gap-value

5. CONCLUSION

The purpose of this paper is to develop a dynamic traffic assignment model for responsive signal control and variable message sign. A mathematical formulation and its solution algorithm are also proposed for the integrated model which is combined with adaptive signal policy and VMS information strategy. This paper showed that solution algorithm developed in the paper converge to Wardrop equilibrium.

The integrated dynamic traffic assignment model in the paper is expected to be useful to develop and evaluate the various ITS' traffic strategies such as signal control and information providing. The model is also expected to enable to simulate the network conditions more precisely than conventional traffic assignment models in that the traffic model will be combined with responsive signal control and route information systems.

Further studies related to this research is to apply the model to real road networks.

REFERENCES

Allsop, R. E. (1971) Delay-minimising settings for fixed time traffic signals at a single road junction. **J. Inst. Math. Its Appl. 8,** 164-85

Allsop, R. E. and J. A. Charlesworth (1977) Traffic in a signal controlled road network. **Traffic Eng.and Control 18**, 262-4

Cantarella, G. E, A.Improta and A.Sforza (1991) Iterative procedure for equilibrium network traffic signal setting.**Trasnportation Research 25A, 241-249**

Fisk, C. S. (1984) Game theory and transportation systems modelling. **Transportation Research, 18B**, 301-313

Gartner, N. H., Gershwin S. B., Little J. D. C. and Ross P. (1980) Pilot study of computer-based urban traffic management, **Transportation Research 14B**, 203-217

Hearn D.W.(1982) The gap function of a convex program, **Operations Researches, Let.1,** 67-71

Janson, B.(1991) Dynamic traffic assignment for urban road networks, **Transportation Research 25B**, 143-161

Marcotte. P. (1983) Network optimization with continuous control parameters. **Transportation Science, 17**, 181-197

Sheffi Y. and Powell W. B. (1983) Optimal signal setting over transportation networks. **Transportation Eng., 109(6)**, 824-839

Smith, M. J. (1979) The existence,uniqueness and stability of traffic equilibria. **Transportation Research 13(B)**, 295-304

Allsop, R. E. (1974) Some possibilities for using traffic control to influence trip destinations and route choice, **Proceedings of the 6th International Symposium on Transportation and Traffic Theory**, 345-374. Elsevier

Lee, S. Lim,K., Kim,G. and Lim,Y.(1996) Assessment of dynamic traffic management schemes of responsive signal control and variable message sign, **the 3rd Intelligent Transport Systems World Congress**, Oct. 14-18, Orlando,USA

Smith, M. J. (1981) A theoretical study of traffic assignment and traffic control. **Proceedings of the 8th International Symposium on Transportation and Traffic Theory**, Toronto.

Smith, M. J., Van Vuren T., Heydecker B. G. and Van Vliet D. (1987) The interaction between signal-control policies and route choice. **Proceedings of the 10th International Symposium on Transportation and Traffic Theory,** 319-338, MIT press

Yang H. and S. Yagar(1994) Traffic assignment and traffic control in general freeway-arterial corridor systems, **Transportation Research(B),** Vol.28B, 463-486

Lim, Y.T.(1997) Development of the dynamic traffic management models with signal control and variable message sign, Ph.D dissertation, Seoul National University

Webster, F. V. (1958) Traffic signal settings. Road Research Technical paper no.39, H.M.S.O.