FORMULATING CUT MATRIX AND OD-CUT MATRIX FOR SENSITIVITY ANALYSIS OF ONE-WAY STREET SYSTEM

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abstract: In this paper we discuss the method of formulating the cut matrix and O-D cut matrix for sensitivity analysis of one-way street system. The effects of comprehensive traffic management on the road network capacity is analyzed by the mathematical programming based on the cut matrix and OD-cut matrix. Sensitivity analysis of the transportation demand management is also performed using simple matrix algebra based on these matrices.

1. INTRODUCTION

As urban areas continue to grow in size, and as road travel continue to increase, traffic congestion will extend over lager areas for longer period of the day. It is necessary to invest sufficiently in highway and street networks that can fully tackle the problem of increasing traffic congestion. However, it becomes difficult to construct new transportation infrastructures for increasing road traffic because of expensive land price and construction costs. Moreover, we do not have enough space to accommodate new transportation facilities in the center of cities even if they are underground. Nowadays, the main target of transportation planning is changing from satisfying traffic demand by enlarging the capacity of infrastructures to controlling traffic demand with supply by comprehensive traffic management of arterial roads or transportation demand management (TDM).

In order to develop a quantitative basis for such progressive approaches, road network capacity is considered as one of the evaluation measures of transportation planning. The road network has a capacity for traffic flow as an upper limit of travel demand, because each node and link in the network has a traffic capacity. The road network capacity is defined as the maximum number of vehicle trips which can be loaded on the road network under the specific restraints: physical and/or environmental constraints. Since road network capacity is defined by network characteristics(road network pattern, link capacity, etc.) and flow characteristics (land use pattern, OD traffic pattern, etc.), it is possible to make balance between traffic demand and road network capacity by means of various improvements based on these two kind of characteristics.

Buchanan(1963) first discussed the maximum capacity of an urban center. The capacity in urban area could be decide by the sums of approaching road capacity. There are a few empirical studies of road network capacity by Smeed(1966), Olszewski and Suchorewski(1987). However, these studies didn't explicitly consider the road system in a city in terms of the road network. On the other hand, in Japan, two different approach to estimate the road network capacity have been studied. One is an application of Linear Programming (Masuya and Kaku 1984) and the other is the traffic assignment simulation method (Iida 1972; Nishimura 1975; Asakura et al 1992; Masuya, Tamura and Saito 1994).

One-way street system is generally used to reduce congestion and to increase the capacity of a street network. In major activity centers, such as the central business district of a city with many high-traffic, closely spaced intersections, one-way system is frequently used because of traffic signal timing considerations and to improve street capacity. In this paper we discuss the method of formulating the cut matrix and OD cut matrix for sensitivity analysis of one-way street system. The effects of comprehensive traffic management on the road network capacity is analyzed by the mathematical programming based on the cut matrix and OD-cut matrix. Sensitivity analysis of the transportation demand management, such as the effects of the change of travel demand pattern on the road network capacity, is also performed using simple matrix algebra based on these matrices.

2. FORMULATION OF CUT MATRIX AND O-D CUT MATRIX

2.1 Estimation of Road Network Capacity

The road network capacity means the maximal acceptable level of traffic flow demand over the network, and it is affected by the link capacity and/or OD traffic pattern over the urban area. The maximal flow is restricted by the minimal capacity of the bottleneck section (minimum-cut) of the network. In advance of model formulation of road network capacity, let us introduce the traffic assignment simulation approach for fixed unit OD pattern. The incremental assignment method is applied, in which small amounts of vehicle trips are gradually load to the network until minimum-cut is searched. The traffic flow pattern calculated is in approximate equilibrium because passing time on the directed link is changed responding to the traffic volume on it at every step of the simulation. Link performance functions are used to represent the effect of congestion on passing time. The calculation steps of road network capacity for one-way street system are shown as follows.

Step. 1 Initialize every directed link flow $V_a^{n,m} = 0$、 the number of iterations n=1 and the number of reload iterations m=0.

Step. 2 Update directed link travel time $t_a^{n,m} = t_a(v_a^{n,m})$, search the minimum path between each OD pair and calculate OD travel time.

Step. 3 Load OD flow , $\Delta T_k = \Delta T \times p_k$, to the minimum path and update directed link flow $V_{a\ k}^{n+1,m} = V_{a\ k-1}^{n+1,m} + \Delta T_k$

Step. 4 If $V_{a\ k}^{n+1,m} \leq C_a$ for all directed links then go to Step. 5, otherwise, go to Step. 6.

Step. 5 Set k=k+1 and return back to Step. 3, if when all OD flow loaded, then set n=n+1 and return back toSstep. 2.

Step. 6 Judge the network connection, if the set of directed links over its capacity make the cut set, then go to Step. 8, otherwise, go to Step. 7.

Step. 7 Set m=m+1, $\Delta T_k = \Delta T_k^m$, $V_a^{n+1,m} = V_{a\ k}^{n+1,m}$ remove the directed links over its capacity and return back to Step. 2.

Step. 8 Calculate the road network capacity RN (Eq.(1) and (2)) and close the calculation

$$P = \left\{ \sum_{a \in T} C_a + \sum_{k \in R} \Delta T_k^m \right\} / n \cdot \Delta T \qquad \text{-------------(1)}$$

$$RN = \sum_{a \in T} C_a / P \qquad \text{-------------(2)}$$

Used the notation are as follows;

$V_a^{n,m}$: traffic flow on directed link a

$t_a^{n,m}$: travel time on directed link a

$t_a(v_a^{n,m})$: link travel time function($= t_{a0}\{1 + r(V_a / C_a)^k\}$)(BPR performance function)

t_{a0}: free-flow travel time

p_k : given unit traffic volumes of k-th OD pair

C_a: capacity of directed link a

ΔT : incremental amount for each iteration

ΔT_k : assigned OD flow of k-th OD pair

ΔT_k^m: unassigned OD flow of k-th OD pair

P : sum of the unit traffic volumes crossing the minimum cut

n : iteration counter

T : set of directed links constitute minimum cut

R : set of OD pairs unassigned by minimum cut

As mentioned in the above, it is made possible to search for a cut (minimum cut) that consists of the directed links, none of which exceed its own capacity based on incremental assignment simulation technique (user equilibrium condition), as well as to estimate the road network capacity.

2.2 Formulation of Cut Matrix and OD-Cut Matrix

Formulating the cut matrix and OD cut matrix is required to quantitatively analyze the effects of comprehensive traffic management on the road network capacity. The cuts with acceptable flow greater than the minimum cut must be produced for formulating these matrix. The incremental assignment method is also applied in which small amounts of vehicle are graduated loaded to the network until traffic volumes reaches its capacity. In making a search for these cuts, various problem arise such as how to assign the traffic demand that might exceed the road network capacity, how to increase the capacity of directed link that constitutes the cut including the minimum cut that may become shortage of capacity.

Since the purpose of this paper is search for the cuts taking the balance between the demand (traffic flow on directed link (traffic demand)) and supply (capacity of directed link in existing road network), the search for the cuts is attempted under the following assumptions: (1) When the total number of trips that exceed the road network capacity are going to be loaded, increasing the capacity is carried out in the cuts whose capacity would become shortage. (2) In other words, in the directed links that constitute minimum cut and cuts whose capacity are not enough, increasing the capacity is to be carried out taking the ratio of traffic flow on directed link on the assumption that total number of trips equivalent to the road network capacity are assigned. (3) And the total number of trips that exceed the road network capacity are assigned to the road network in which the directed links that constitute the cuts whose capacity is insufficient have been increased. Then the algorithm of formulating the cut matrix and OD cut matrix for one-way street system is shown as follows.

Step. 1 Load the traffic demand (OD flow) equivalent to the road network capacity RN (Eq. (2)) on to the network by incremental assignment technique and calculate the ratio of traffic flow W_a on directed link from the traffic flow V_a (Eq.(3))

$$W_a = V_a / RN \qquad \text{-------------(3)}$$

Step. 2 Increase the capacity $\Delta C_a^1 \cdot$ (Eq. (4)) for the directed links that constitute the minimum cut respectively to load the extra number of trips Δ T over the road network which the traffic demand RN have assigned and the number of iteration n=1

$$\Delta C_a^1 = n \cdot \Delta T \cdot W_a \qquad \text{-------------(4)}$$

Step. 3 Carry out the calculation procedures Step. 2 ~7 in Section 2.1 to load OD flow $\Delta T_k = \Delta T \times p_k$, to the minimum path and update the directed link flow

Step. 4 Judge the network connection, if the set of directed links over its capacity make the cut set during the procedure Step. 3, then calculate the acceptable flow F_i of its cut (Eq.(5))

$$F_i = \sum_{a \in T_i} C_a / \sum_{a \in T_i} W_a \qquad \text{-------------(5)}$$

where, T_i ; the set of directed links constitute the cut i

Step. 5 Increase the capacity ΔC_a^i (Eq.(6)) for the directed links that constitute the cut i and return back to Step. 3 to load OD flow that has not yet been assigned by cut i

$$\Delta C_a^i = (RN + n \cdot \Delta T - F_i) \cdot W_a \qquad \text{-------------(6)}$$

Step. 6 Repeat Step. 3~ 5 until all OD flow has been loaded, if all OD flow has been loaded, then go to Step. 7

Step. 7 Repeat Step. 3~6 to load incremental amount Δ T and set n=n+1

Step. 8 Repeat Step. 3~7 until a certain number of trips (n・Δ T) is reached considering the necessary number of cuts for the sensitivity analysis

Step. 9 Search for other elementary cuts from the combination of independent cuts explored in Step. 1~8

Step. 10 Determine the directed links which constitute every cuts searched for in Step. 1～9 and finally formulate the cut matrix **C**. Where, if the directed link j constitute the cut i and its direction is the same as cut i then $c_{ij}=1$, if the directed link j constitute the cut i and its direction is opposite to cut i then $c_{ij}=-1$, and otherwise $c_{ij}=0$.

Step. 11 Determine which cut section each OD pair would cross among the cuts searched for in Step. 1～9 and finally formulate the OD-cut matrix **K**. Where, if OD pair j cross the cut i , then $k_{ij}=1$, and otherwise $k_{ij}=0$.

Allowing for all combinations in Step. 9, in case of W number of cuts, there exists 2^{W-1} different combinations; thus requiring a large number of calculations. However, by dividing the links constituting the road network into either those bordering on only the inner region or those in both inner and outer regions as well as by introducing Graph Theory, i.e., star without loop is an elementary cut, a search for elementary cuts can be easily be conducted.

2.3 Algorithm for Judgment of Network Connection

Road network including the one-way street system is represented as a directed network. In a directed network case, it is difficult to judge the network connection compared to a undirected network. In this paper we develop a algorithm which can be judged the network connection of a directed network. Its algorithm is based on the network theory that if the directed network is divided into two exclusive subnetwork, then OD flow can not be assigned more to the directed network against the generated cut set which is the set of directed links over its capacity. Algorithm for judgment of network connection can be summarized as follows:

Step. 1 Let **D** be node-link incidence matrix. Where if the directed link j incident out of the origin node i , then $d_{ij}=1$, if the directed link j incident into the destination node i, then $d_{ij}=-1$, and otherwise $d_{ij}=0$.

Step. 2 Enumerate the unassigned OD pairs because of the generated cut set and divide the nodes of each OD pairs into the origin node and the destination node.

Step. 3 Let **N** be node vector. Where the element =1 if node i is the origin node, and all other elements (node i is the destination node) =0

Step. 4 Let **E** be the directed link vector obtained by multiplying node-link incidence matrix **D** by node vector **N**. Where the element = 1 or -1 if the directed link compose the cut set, and all otherwise elements =0.

$$\mathbf{E}=\mathbf{N}\cdot\mathbf{D} \qquad \text{-------------(7)}$$

Step. 5 Judge the network connection whether the set of directed links that take 1 or -1 in Step. 4 constitute the cut set or not.

It is made possible to search for the directed links which constitute the cut set, that is , make the network divide into two exclusive subnetwork , by using simple matrix algebra.

2.4 Numerical Example

To illustrate the discussion in the previous Section, let us consider the directed road network (10 nodes and 24 directed links (include 6 one-way streets)) depicted in Fig. 1. Table 1 lists the unit OD traffic volumes (upper right half) and the directed link length (lower left half). The directed link number is given above links in Fig. 1. Here, Six directed links (link number 1 to 6) are one-way streets. Let the number of traffic lanes of each directed link be one and capacity of each directed link be 1800. The capacity of one-way street is 4320 of a 20% increase over the other directed link. The parameters of link travel time function (BPR function) are as follows: r=2.62, k=5.

Calculation of the road network capacity gives 20366 (=6120/0.3005) according to the procedure in Section 2.1. This value is equivalent to the acceptable flow of cut 1(link 1,7)and cut 2 (link 5,8) (minimum cut) in Fig. 2. The direction of cut 2 is opposite to cut 1. Traffic flow and ratio of flow on the directed link are shown in Table 2. In case of the directed network, the difference of traffic flow and ratio of flow, such as link 7 and 8,

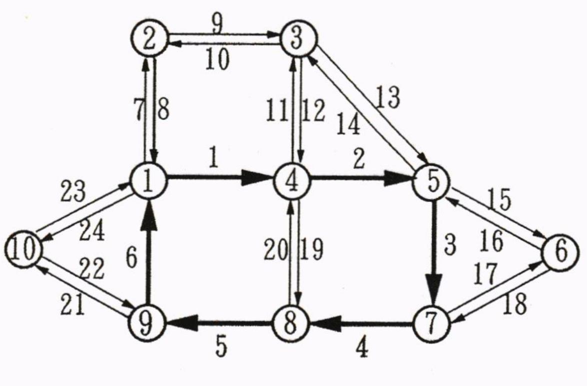

Fig. 1 One-way street system

Table 1 Unit OD traffic volumes and directed link length(m)

O D	1	2	3	4	5	6	7	8	9	10	sum
1	0	0.0475	0.0385	0.0475	0.0420	0.0520	0.0280	0.0070	0.0095	0.0025	0.2745
2	500	0	0.0190	0.0110	0.0060	0.0065	0.0030	0.0000	0.0120	0.0005	0.1055
3	∞	300	0	0.0250	0.0080	0.0075	0.0025	0.0025	0.0000	0.0015	0.1045
4	∞	∞	500	0	0.0145	0.0110	0.0015	0.0020	0.0100	0.0005	0.1230
5	∞	∞	600	300	0	0.0350	0.0065	0.0015	0.0000	0.0000	0.1135
6	∞	∞	∞	∞	300	0	0.0160	0.0015	0.0020	0.0010	0.1325
7	∞	∞	∞	∞	400	300	0	0.0065	0.0025	0.0005	0.0670
8	∞	∞	∞	400	∞	∞	300	0	0.0055	0.0005	0.0270
9	400	∞	∞	∞	∞	∞	∞	300	0	0.0020	0.0435
10	300	∞	∞	∞	∞	∞	∞	∞	300	∞	0.0090
sum											1.0000

Table 2 Traffic flow and ratio of flow on the directed link

arc#	traffic flow	ratio of flow	arc#	traffic flow	ratio of flow
1	4320	0.2121	13	847	0.0416
2	3263	0.1602	14	980	0.0481
3	2672	0.1312	15	1678	0.0824
4	3130	0.1537	16	1224	0.0601
5	4320	0.2121	17	1244	0.0611
6	3546	0.1741	18	1701	0.0835
7	1800	0.0884	19	1800	0.0884
8	1800	0.0884	20	611	0.0300
9	1635	0.0803	21	1198	0.0588
10	1635	0.0803	22	4256	0.2090
11	1187	0.0583	23	1238	0.0608
12	1320	0.0648	24	466	0.0229

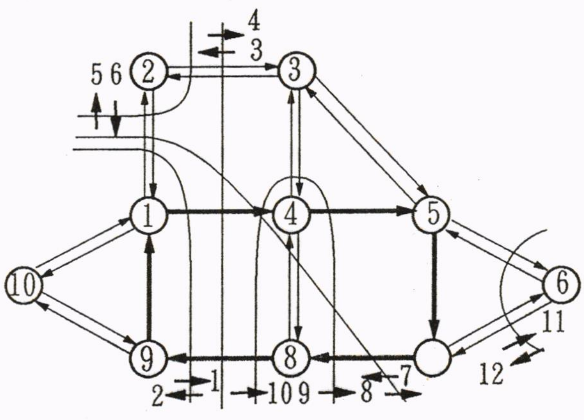

Fig. 2 Search for independent and elementary cuts

depends on direction. A search is conducted for the 6 independent cuts as shown in Fig. 2 according to the procedure in Section 2.2. Furthermore, a search is made for other elementary cuts based on the combination of independent 6 cuts (Fig. 2).

Table 3 Capacity of cut , sum of the unit OD traffic volumes crossing the cut and acceptable flow

cut#	C_i	P_i	F_i	P'_i	$P_i - P'_i$
1	6120	0.3005	20366	0.2990	0.0015
2	6120	0.3005	20366	0.2990	0.0015
3	6120	0.2924	20930	0.2845	0.0079
4	6120	0.2924	20930	0.2845	0.0079
5	3600	0.1687	21340	0.1055	0.0632
6	3600	0.1687	21340	0.1055	0.0632
7	7920	0.3305	23964	0.3000	0.0305
8	7920	0.3305	23964	0.3000	0.0305
9	10440	0.4306	24245	0.1460	0.2846
10	10440	0.4306	24245	0.1460	0.2846
11	7920	0.3224	24566	0.2855	0.0369
12	7920	0.3224	24566	0.2855	0.0369

cut \ arc	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	0	0	0	-1	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	-1	0	0	0	1	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	-1	0	0	0	1	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	0	0	0	-1	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	-1	0	0	1	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0
8	1	0	0	-1	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0
9	-1	1	0	-1	1	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
10	1	-1	0	1	-1	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
11	-1	0	0	1	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0

Fig. 3 Cut matrix **C**

1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0
2	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0
3	1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0
4	1	1	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0
5	1	0	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0
6	0	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0
7	0	1	0	1	0	1	0	1	0	1	0	1	1	0	0	0	0	0
8	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0
9	1	1	1	1	0	1	1	1	1	1	1	1	0	1	0	0	0	0
	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	0	0	0
	1	1	0	1	0	1	0	1	0	1	0	1	1	0	0	0	0	0
	1	1	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0

Fig. 4 OD-cut matrix **K**

In regard to these elementary cuts, the capacity of the cut (C_i), sum of the unit OD traffic volumes crossing the cut (P_i), and acceptable flow (F_i) are shown in Table 3. Here P'_i is sum of the unit OD traffic volumes which can not be assigned more to the directed network against the generated cut set i. Therefore, it is possible to make the assigned route of the OD traffic volumes ($P_i - P'_i$) alter to the alternative route that does not cross the cut i. OD traffic volumes are assigned based on user equilibrium condition. By putting the cutsin order of acceptable flow (from small to large), the cut matrix **C** and OD-cut matrix **K** are obtained (Fig. 3 and 4). The OD pairs illustrated in Fig. 4 are only those generated and attracted to node 1. The an acceptable flow of 11 cuts represented in Fig 2 are smaller than 25000 and these cuts are also required for the analysis as given in latter Section.

3. SENSITIVITY ANALYSIS BASED ON CUT MATRIX AND OD-CUT MATRIX

Since road network capacity is defined by network characteristics (road network pattern, link capacity, etc.) and flow characteristics (land use pattern, OD traffic pattern, etc.), it is possible to make balance between traffic demand and road network capacity by means of various improvements based on these two kind of characteristics. The improvements based on the network characteristics are those of widening lane widths or lateral clearance, signal coordination, parking controls and so on. On the other hand, improvements based on the flow characteristics are those of altering the route by driver information system or various transportation demand management, such as changing the mode of travel, land-use control by decreasing the need for travel, discouraging vehicle trips in the peak hour and so on. (ITE 1989; Underwood 1993, Kashiwadani 1996)

3.1 Sensitivity Analysis of Comprehensive Traffic Management

Comprehensive traffic management of arterial roads is to provide for the safe, orderly and efficient movement of persons and goods, and to maximize the use of existing facilities. Here, we consider both widening of lane widths or lateral clearance as network characteristics and altering the route by driver information system as flow characteristics. A sensitivity analysis is carried out by formulating the mathematical programming based on the cut matrix and OD-cut matrix.

Now, let W be the number of cuts to be made in the formulation. The linear programming problem for sensitivity analysis is formulated as follows.

$$\text{Maximize} \quad RC \qquad \text{-------------(8)}$$

subject to

$$\sum_{r=1}^{n_k} Y_r^k = p_k \cdot RC \quad (k \in K) \qquad \text{-------------(9)}$$

$$\sum_{k\in K}\sum_{r=1}^{n_k} {}_w\delta_r^k \cdot Y_r^k + (P_w - \sum_{k\in K} {}_w\delta_r^k \cdot Y_r^k p_w^k) \cdot RC \le \sum_{i\in R_w} C_a \cdot x_a + C_w \quad (w = 1,2,\cdots,W) \text{----- (10)}$$

$$\sum_{a=1}^{m} C_a \cdot x_a \le TC \qquad \text{-------------(11)}$$

$$x_a \ge 0 \quad (a = 1,2,\cdots,m) \qquad \text{-------------(12)}$$

$$Y_r^k \ge 0 \quad \begin{Bmatrix} k \in K \\ r = 1,2,\cdots,n_k \end{Bmatrix} \qquad \text{-------------(13)}$$

Where

Y_r^k : traffic volumes of k-th OD pair on path r

n_k : number of independent paths of k-th OD pair

K: set of OD pairs altered the route

${}_w\delta_r^k$: if directed link i is on path r of OD pair k, then =1, otherwise =0

p_w^k : unit OD traffic volumes of k-th OD pair crossing cut w

x_a : improvement made to directed link a (meters of road widening)

TC : total investment meters of road widening

These procedure are based on the assumption that improvement to the directed links increases the capacity of cut and altering the route of OD pairs decreases sum of unit OD traffic volumes of the cut which the OD pair in question crosses; thus also increasing the acceptable flow of the cut and the road network capacity. A sensitivity analysis of comprehensive traffic management can be accomplished by calculating a parametric linear programming while increasing TC as parameter step by step.

For example, if TC increases 500 by 500 from 0, the effects of the road network capacity by only improvement the direct links is estimated in Table 4 and Fig. 5. Furthermore, the road network capacity considering flow characteristic (altering the route of OD pairs) is also shown in Table 4 and Fig. 5. Here, OD pairs which are imposed on altering to the other route are OD1-2、2-1、1-3、3-1、2-4, 7-2 and so on. These OD pairs correspond to OD traffic of ($P_i - P'_i$) shown in Table 4. and cross the cuts that whose acceptable flow is small including minimum cut (cut 1 and cut 2). It is possible to increase the road network capacity by such this comprehensive traffic management.

Table 4 Road network capacity by widening and altering

TC	RC by widening	RC by widening and altering
0	20366	20468
500	21198	21317
1000	22029	22153
1500	22767	22990
2000	23424	23826
2500	24081	24663
3000	24737	25499

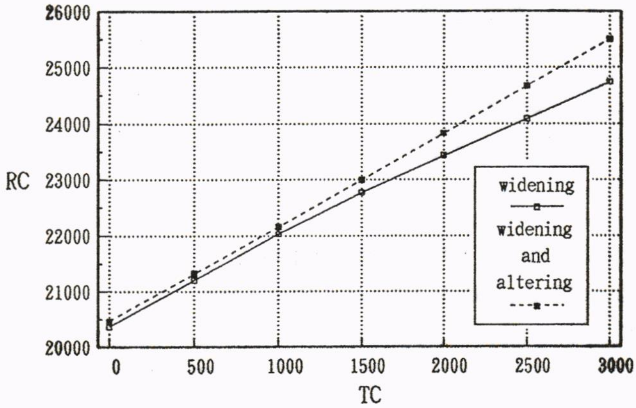

Fig. 5 Road network capacity by widening and altering

3.2 Sensitivity Analysis of Transportation Demand Management

Various TDM alternatives , in other words, attempt to increase the road network capacity encouraging changes in the OD traffic pattern. Therefore, we consider effects on the road network capacity brought about by changes in the OD traffic pattern. A sensitivity analysis is carried out by calculating the road network capacity accompanying changes in the OD traffic pattern via simple matrix operation based on the proposed cut matrix and OD-cut matrix.Now, let W be the number of cuts to be made in the calculation. Then, the steps required to calculated the road network capacity accompanying changes in the OD traffic pattern are as follows:

Step 1. Let **L** be column vector for the capacity of each links (Eq.(14)). The capacity of each cut M_w is obtained by multiplying column vector **L** by cut matrix **C**. Where **L** is the M×1 column vector, **C** is the w×m matrix and super script t denotes matrix transposition.

$$\mathbf{L}=[L_1, L_2, \cdots\cdots, L_l, \cdots\cdots, L_m]^t \qquad \text{-------------(14)}$$

$$\mathbf{M}=\mathbf{C}\cdot\mathbf{L} \qquad \text{-------------(15)}$$

Step 2. Let **p** be column vector for the unit OD traffic volumes (Eq.(16)). Replace each element of column vector **p** by the unit OD traffic volumes resulting from changes in the OD traffic pattern due to various TDM alternatives. Then, the sum of the unit OD traffic volumes through the cut (P_w) is obtained by multiplying column vector **p** by the OD-cut matrix **K**. Where **p** is the r×1 column vector, **P** is the w×1 column vector and **K** is the w×q(the number of OD pairs) matrix.

$$\mathbf{p}=[p_1, p_2, \cdots\cdots, p_k, \cdots\cdots, p_r]^t \qquad \text{-------------(16)}$$

$$\mathbf{P}=[P_1, P_2, \cdots\cdots, P_w, \cdots\cdots, P_W]^t=\mathbf{K}\cdot\mathbf{p} \qquad \text{-------------(17)}$$

Step 3. Substitute the inverse values of the P_w elements of column vector **P** for the diagonal elements of matrix **R**. Where **R** is the w×w matrix and $R_{ww}=1/P_w$.

$$\mathbf{R}=\begin{bmatrix} R_{11} & & & 0 \\ & R_{22} & & \\ & & \ddots & \\ 0 & & & R_{ww} \end{bmatrix} \qquad \text{-------------(18)}$$

Step 4. Calculation the acceptable flow of each cut (Fw) by multiplying column vector **M** by matrix **R**. Where **F** is the w×1 column vector.

$$\mathbf{F}=\mathbf{R}\cdot\mathbf{M} \qquad \text{-------------(19)}$$

Step 5. The road network capacity RN accompanying changes in the OD traffic pattern can be obtained using a cut with the minimum acceptable flow.

$$RN=\min[F_1, F_2, \cdots\cdots\cdots\cdots, F_W] \qquad \text{-------------(20)}$$

These procedure are based on the assumption that changes in the unit OD traffic volumes alters the acceptable flow of the cut which the OD pair in question crosses; thus also changing the road network capacity. A sensitivity analysis of the accompanying changes in the OD traffic pattern can be accomplished by comparing the capacity of the existing road network and RN of Eq.(20).

Table 5 Unit OD traffic volumes of patter 2 and pattern 3

	1	2	3	4	5	6	7	8	9	10	sum
1		0.0404	0.0327	0.0404	0.0357	0.0442	0.0238	0.0060	0.0081	0.0020	0.2333
2	0.0534		0.0190	0.0110	0.0060	0.0065	0.0030	0.0000	0.0120	0.0005	0.0984
3	0.0433	0.0162		0.0375	0.0120	0.0113	0.0038	0.0038	0.0000	0.0023	0.1224
4	0.0536	0.0094	0.0220		0.0160	0.0121	0.0017	0.0022	0.0110	0.0006	0.1325
5	0.0473	0.0051	0.0070	0.0116		0.0455	0.0085	0.0020	0.0000	0.0000	0.1257
6	0.0583	0.0055	0.0066	0.0088	0.0116		0.0160	0.0015	0.0020	0.0010	0.1401
7	0.0315	0.0026	0.0022	0.0012	0.0088	0.0160		0.0065	0.0025	0.0005	0.0663
8	0.0079	0.0000	0.0022	0.0016	0.0012	0.0015	0.0065		0.0050	0.0006	0.0276
9	0.0108	0.0102	0.0000	0.0080	0.0016	0.0020	0.0025	0.0055		0.0028	0.0434
10	0.0029	0.0004	0.0013	0.0004	0.0080	0.0005	0.0005	0.0005	0.0020		0.0103
sum	0.309	0.1028	0.1008	0.1166	0.1022	0.1108	0.0718	0.0269	0.0426	0.0165	1.0000

Table 6 sum of unit OD traffic volumes crossing the cut and acceptable flow

		pattern 1		pattern 2		pattern 3	
cut#	C_i	P_i	F_i	P_i	F_i	P_i	F_i
1	6120	0.3005	20366	0.2625	23313	0.3384	18086
2	6120	0.3005	20366	0.2625	23313	0.3384	18086
3	6120	0.2924	20930	0.2608	23462	0.3201	19116
4	6120	0.2924	20930	0.2608	23462	0.3201	19116
5	3600	0.1687	21340	0.1573	22879	0.1644	21900
6	3600	0.1687	21340	0.1573	22879	0.1644	21900
7	7920	0.3305	23964	0.2926	27067	0.3699	21409
8	7920	0.3305	23964	0.2926	27067	0.3699	21409
9	10440	0.4306	24245	0.4592	22735	0.4138	25230
10	10440	0.4306	24245	0.4592	22735	0.4138	25230
11	7920	0.3224	24566	0.2916	27163	0.3507	22581
12	7920	0.3224	24566	0.2916	27163	0.3507	22581

For example, if OD traffic pattern changes from pattern 1 (already calculated in Section 2.4) to pattern 2 (upper right half) or pattern 3 (lower left half) listed in Table 5, the effects of the road network capacity by these changes is estimated according to the above procedure. In pattern 2 case, the sum of the unit OD traffic volumes through the cut 1 and cut 2 (minimum cut represented in Fig. 2) is decreased by changes in the OD traffic pattern. On the other hand, in pattern 3 case, the sum of the unit OD traffic volumes through the cut 1 and cut 2 is increased. Table 6 shows the result of acceptable flow of each cut accompanying changes in the OD traffic pattern. The minimum cut of pattern 2 turn into cut 9 and cut 10 and the road network capacity is increased its acceptable flow (22735). In pattern 3 case, the minimum cut is cut 1 and cut 2 as same as pattern 1. However the road network capacity (18086) is decreased because of the increase of the sum of the unit OD traffic volumes through the cut 1 and cut 2. Therefore, changes in OD traffic pattern which decrease the sum of the unit OD traffic volumes through the cut of the smaller acceptable flow, such as cut 1, cut 3 or cut 5, increase the road network capacity, but the contrary change pattern, such as pattern 3, decrease the road network capacity.

4. CONCLUSIONS

In the present study, the method of formulating the cut matrix **C** and OD-cut matrix **K** for sensitivity analysis of one-way street system is based on the Incremental Assignment Simulation technique. Since road network capacity is defined by network characteristics and flow characteristics, it is possible to make balance between traffic demand and road network capacity by means of various improvements based on these two kind of characteristics. The improvements based on the network characteristics are those of widening lane widths or lateral clearance, signal coordination, parking controls and so on. On the other hand, improvements based on the flow characteristics are those of altering the route by driver information system or various trasportation demand management, such as changing the mode of travel, land-use control by decreasing the need for travel, discouraging vehicle trips in the peak hour and so on. The effects of comprehensive traffic management on the road network capacity is analized by the mathematical programming based on the cut matrix and OD-cut matrix. Sensitivity analysis of the transportation demand management, such as the effects of the change of travel demand pattern on the road network capacity, is also performed using simple matrix algebra based on these matrices. In the future, research will be conducted concerning the extent various TDM alternatives influence on the OD traffic pattern. Since enlargements of the road network rapidly increase the number of OD pairs and cuts, the study will include applications to actual one-way street system.

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