COMPARISON OF SOME RELIABILITY MODELS IN A DETERIORATED ROAD NETWORK

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abstract: This paper compares some reliability models for evaluating a road network deteriorated by natural disasters. The User Equilibrium traffic assignment with variable demand and strict link capacity constraint is applied for calculating travel times, travel demand and consumer's surplus between an OD pair of the network with some degraded links. Reliability is defined as the probability of whether one of those performance measures is kept within an acceptable level, and an approximation algorithm common for calculating reliability models is proposed. Numerical features of those reliability models are compared through small scale examples.

1. INTRODUCTION

Flows in a transport network are interpreted as the results of interaction between travel demand and supply conditions of the network. Even if all of the links and nodes of a network are not physically degraded and the condition of the network is usual, flows in the network may not always be stable. Because travel demand is fluctuating from time-to-time and day-by-day. We sometimes face unusual network flow conditions due to seasonal fluctuations of travel demand.

When some links of a network are closed by traffic accidents, roadworks or natural disasters, flows in the network will become more unstable. Almost all of the links are damaged in an extraordinary natural disaster such as the great Hanshin Earthquake. We will suffer from extremely heavy traffic congestion caused by the interaction between travel demand and degraded network capacity. Figure 1 shows a conceptual relation of the physical condition of a road network and traffic flows in the corresponding network. Describing traffic flows and the level of service and estimating network performance in deteriorated conditions are necessary for the strategic transport network planning under uncertainty.

A reliable network means a network which can guarantee an acceptable level of service for road traffic if the functions of some links of the network are degraded by disasters. Network reliability models have been studied for evaluating road networks in both usual and unusual conditions. Turnquist and Bowman(1980) presented a set of simulation experiences to study the effects of the structure of the urban network on service reliability. Iida and Wakabayashi(1989) developed an approximation method of calculating the connectivity between a node pair in a network. These studies are concerned with the reliability analysis of a *pure network* and flows in the network were not explicitly considered.

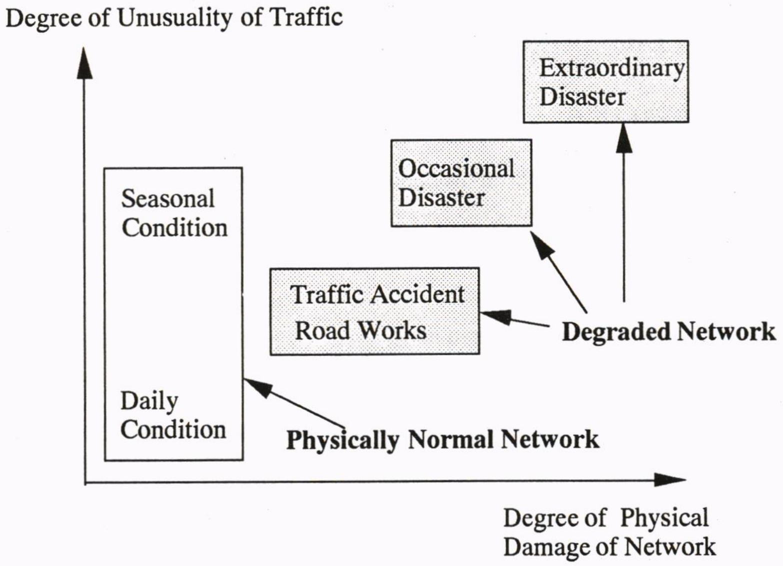

Figure 1 Level of Network Degradation and Corresponding Flow Conditions

Asakura and Kashiwadani(1991) proposed a time reliability model of a network considering day-to-day fluctuation of traffic flow. Although they used a traffic assignment model, it was calculated independently from network conditions. Du and Nicholson(1993) showed a general framework of the analysis and design of Degradable Transportation Systems. The User Equilibrium assignment was involved in the reliability analysis. By extending the algorithm shown by Du and Nicholson, Asakura(1996) presented an approximation algorithm of the distribution function of a performance measure in a deteriorated network. These studies focus on a *flow network*, in which the interaction between travel demand and network condition is described. However, previous studies were not sufficient for discussing the relation between different reliability models.

A flow network should be studied for evaluating the reliability of a degraded road network, in which the inconvenience of travel may bring the reduction of travel demand and network flow pattern may change. This paper aims to compare some different reliability models of an origin and destination (OD) pair in a road network when some links are possibly damaged by natural disasters and may be closed to traffic. Travel time, travel demand and consumer's surplus between an OD pair are respectively used as performance measures in those reliability models.

In the following Chapter 2, the Network User Equilibrium model with variable demand and strict link capacity constraints is applied to describe flows in a network with some disconnected links. Chapter 3 shows an approximation algorithm for estimating the distribution functions of performance measures between an OD pair. Numerical examples are calculated in Chapter 4 to compare reliability models proposed. The effects of network structure and congestion level on network reliability are studied through those examples.

2. FLOWS IN A DETERIORATED NETWORK

2.1 State Vector and State Probability

A road network is represented by a directed graph which consists of a set of numbered nodes and a set of numbered links. We first assume that links may be degraded by disasters, while nodes will not be. If it is necessary to consider the failure of a node, it is represented in detail using several links which may become incapable. Thus, the first assumption above will not lose generality.

In order to describe the flow in a degraded network as simply as possible, we assume that a deteriorated link is completely closed to traffic and that the failed condition of a link continues for a long time, at least the expected longest travel time of origin-destination (OD) pairs in the network. A link may be partially closed and the condition of a network may change quickly in actual degraded situations. However, allowing for this makes it difficult to calculate the flow in a deteriorated network.

A degraded road network is identified by the state vector $\mathbf{x}=\{x_1,..,x_a,..,x_L\}$ whose element x_a denotes whether link a is degraded or not; namely x_a is equal to 0 if it is physically damaged and closed to traffic, or x_a is equal to 1 if the link maintains its functions in the ordinary condition. The possible maximum number of the state vector is 2^L, and $X=\{\mathbf{x}\}$ denotes the state vector space. If any link of the network is not degraded, the state vector is written as $\mathbf{x}_0=\{1,...,1\}$. This state is referred as the normal or ordinary state in the following part of this paper. The worst state vector is written as $\mathbf{x}_w=\{0,...,0\}$ in which all links are failed.

Here we define the probability of the occurrence of a state $\mathbf{x}$, $p(\mathbf{x})$. Disasters such as earthquakes are not predictable and the degree of external force due to disaster is quite uncertain. Then, a road link may be durable against natural disasters or may not be. This means the physical conditions of a link are not deterministic. We define p_a as the probability of whether the link a is not degraded and not closed to traffic. Although it is very important to estimate the value of p_a, we assume the value of p_a is exogenously given and fixed. When the probability p_a is independent, the probability of the occurrence of a state $\mathbf{x}$ is calculated as:

$$p(\mathbf{x}) = \prod_{a \in A} p_a^{\,x_a} (1-p_a)^{1-x_a} \tag{1}$$

We call $p(\mathbf{x})$ as state probability.

2.2 Variable Demand User Equilibrium with Link Capacity Constraint

This section shows how to calculate traffic flow in a network of state $\mathbf{x}$ with some degraded links. Compared with the network flow analysis for the normal network condition, the following problems should be considered.

First, the number of available routes between an OD pair will be limited in a degraded network. There may be no available routes in the network. Then, travellers may cancel their trip when they expect longer diversion in a deteriorated network. This will result in the reduction of travel demand. Secondly, some links in a network are closed to traffic and the remaining links become congested, because travellers are concentrated into surviving routes in the deteriorated network. However, each link has its own capacity and the link traffic volume cannot exceed that capacity. A network assignment model considering the first point was proposed by Asakura and Kashiwadani (1995), however they did not discuss link capacity constraint.

The second point means that explicit link capacity constraint must be considered and a link cost function such as the BPR function would be inappropriate. This constraint also excludes considering traffic assignment methods with fixed OD demand. A feasible network flow may not exist if the link capacity is not sufficient for travel demand. The most extreme case is the one in which there is no route available between OD pairs. The fixed demand cannot be loaded onto the network in this case. Thus, we employ traffic assignment methods with variable OD demand, in which OD travel demand is determined as a function of the level of service of the OD pair.

The User Equilibrium assignment with variable OD demand and explicit link capacity constraint seems one of the simplest assignment methods which satisfies the above conditions. As we have already assumed, degraded links would be completely closed to traffic for a long time and the same network state would be held sufficiently long time. Thus, the network users will experience the same network state for a long time and the User Equilibrium condition seems appropriate to describe users' behaviour in a deteriorated network. The UE assignment model with variable demand and link capacity constraint is formulated as follows:

$$Z=\sum_{a\in A}\int_0^{f_a} t_a(x)dx - \sum_{r\in R}\sum_{s\in S}\int_0^{q_{rs}} D_{rs}^{-1}(y)dy \rightarrow \text{minimize}$$

subject to

$$\begin{aligned} &\sum_{k\in K_{rs}} h_k^{rs}=q_{rs} && \forall r\in R, s\in S \\ &h_k^{rs}\geq 0 && \forall k\in K_{rs}, r\in R, s\in S \\ &q_{rs}\geq 0 && \forall r\in R, s\in S \\ &f_a\leq C_a && \forall a\in A \end{aligned} \tag{2}$$

where q_{rs} is the number of trips between OD pair r-s, h_k^{rs} is the flow on the k-th path between OD pair r-s, f_a is the traffic volume of link a and satisfies $f_a=\sum_{r\in R}\sum_{s\in S}\sum_{k\in K_{rs}} h_k^{rs}\, d_{k,a}^{rs}$. $t_a(x)$ is the link cost function with explicit capacity constraint, $D_{rs}^{-1}(y)$, is the inverse of the demand function associated with OD pair r-s.

Davidson's function(1966) is a typical link cost function. That is,

$$t_a(f_a)=t_{a0}\{1+\kappa\, f_a/(C_a-f_a)\} \quad \text{for} \quad 0\leq f_a\leq C_a \tag{3}$$

where κ is a parameter of the function.

The travel demand of an OD pair is assumed to be decreasing for the travel time. An exponential function is employed for the demand function of an OD pair. That is,

$$D_{rs}(t_{rs}) = D^0_{rs}\exp(-\gamma(t_{rs} - t^0_{rs})) \tag{4}$$

where D^0_{rs} and t^0_{rs} denote the upper limit of demand and the free flow travel time of OD pair r-s. γ is a parameter of the demand function.

The formulated UE model includes explicit link capacity constraint, and this is the difference from other UE models with variable demand. The equivalency conditions of the formulated optimization problem and the UE conditions can be proved using the first order necessity conditions of the optimization problem. The existence and uniqueness conditions of the optimal solution can be discussed with the similar method of Patriksson(1994).

2.3 Solution Algorithm

This model can be reformulated as the fixed demand model using the excess demand formulation (Sheffi, 1985).

$$Z = \sum_{a\in A}\int_0^{f_a} t_a(x)dx + \sum_{r\in R}\sum_{s\in S}\int_0^{e_{rs}} W_{rs}(y)dy \rightarrow \text{minimize}$$

subject to

$$\begin{aligned}
&\sum_{k\in K_{rs}} h^{rs}_k + e_{rs} = D^0_{rs} && \forall r\in R, s\in S \\
&h^{rs}_k \geq 0 && \forall k\in K_{rs}, r\in R, s\in S \\
&e_{rs} \geq 0 && \forall r\in R, s\in S \\
&f_a \leq C_a && \forall a\in A
\end{aligned} \tag{5}$$

where e_{rs} denotes the flow of the dummy link connecting the OD pair r-s directly. The link carries only the excess demand flow and capacity constraint is not applied to the execss link. $W_{rs}(y)$ is the argument-complementing function of the inverse demand function, that is $W_{rs}(y) = D^{-1}_{rs}(D^0_{rs} - y)$. This function has the similar functional shape of the link cost function $t_a(x)$ and is increasing for the flow of the excess link. Thus, the reformulated problem yields to the UE problem with fixed demand.

The fixed UE problem with link capacity constraint can be solved using Daganzo's algorithm(1977). The original Daganzo's algorithm requires a large number of iterations for calculating initial feasible link flows within link capacity constraints. However, calculating initial feasibile solution of the reformulated UE problem requires just one iteration since the network includes excess links without capacity. The solution algorithm is shown as follows;

Step.0: *Initialization.* Load the upper limit of each OD demand D^0_{rs} to the excess link r-s, respectively. This makes the initial flows of actual links $\{f^0_a = 0\}$ and those of excess links $\{e^0_{rs} = D^0_{rs}\}$. Set iteration counter n=0.

Step.1: *Travel time update.* Set travel times of actual links $t_a^n = t_a(f_a^n)$ and those of excess links $w_{rs}^n = W_{rs}(D_{rs}^0 - e_{rs}^n)$.

Step.2: *Direction finding.*

(i) Compute the shortest path and travel time u_{rs}^n in the actual network.

(ii) Assign D_{rs}^0 to the shortest path in the actual network if $u_{rs}^n \le w_{rs}^n$, and calculate auxiliary flows y_a^n of actual links.

(iii) If $u_{rs}^n > w_{rs}^n$, load D_{rs}^0 to the excess link and calculate auxiliary flows $z_{rs}^n = D_{rs}^0$.

Step.3: *Move-size determination.* Find α^n that solves

$$\min. \sum_a \int_0^{f_a^n + \alpha(y_a^n - f_a^n)} t_a(x)dx + \sum_{r,s} \int_0^{e_{rs}^n + \alpha(z_{rs}^n - e_{rs}^n)} W_{rs}(y)dy$$

sub. to

$$0 \le \alpha \le \min_a . \{(C_a - f_a^n)/(y_a^n - f_a^n)\}$$

Step.4: *Flow update.* Set

$$f_a^{n+1} = f_a^n + \alpha^n (y_a^n - f_a^n) \qquad \forall a$$

$$e_{rs}^{n+1} = e_{rs}^n + \alpha^n (z_{rs}^n - e_{rs}^n) \qquad \forall r,s$$

Step.5: *Convergence criterion.* If $\sum_a |f_a^{n+1} - f_a^n| / f_a^n + \sum_{r,s} |e_{rs}^{n+1} - e_{rs}^n| / e_{rs}^n \le \varepsilon$, stop. Otherwise, set n=n+1 and go to step1.

In the move-size determination step, the upper limit of α^n must be required so that the flow of the actual links is kept within the capacity constraint.

3. RELIABILITY MODELS

3.1 Performance Indexes of a Flow Network

When we solve the UE problem with variable demand for the network at state $\mathbf{x}$, travel times as well as flow variables in the network are simultaneously calculated. Among those variables, equilibrium flows of OD pair r-s, $q_{rs}(\mathbf{x})$, and equilibrium travel time of OD pair r-s, $u_{rs}(\mathbf{x})$, can be used for evaluating the performance of a deteriorated network. Figure 2 shows the shift of the equilibrium point from the normal state to a deteriorated state of a network. In the deteriorated network, link cost functions conceptually move upwards. Travel times will generally become worse than the normal state and travel demand will decrease due to the reduced level of service.

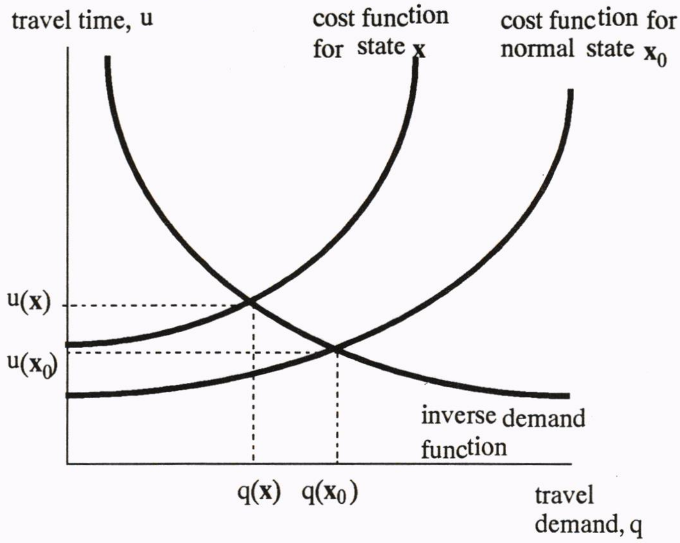

Figure 2 Shift of UE Point

A simple performance index is the travel time of an OD pair. When the OD travel time of a state $\mathbf{x}$ is larger than that of the normal state $\mathbf{x}_0$, the network performance is decreased. The travel time of any OD pair is generally different from those of the others. In order to compare the performance index between different OD pairs, we use the ratio of the OD travel time of a state $\mathbf{x}$ to that of the normal state $\mathbf{x}_0$. That is denoted by $u_{rs}(\mathbf{x})/u_{rs}(\mathbf{x}_0)$.

The OD travel demand may decrease when some links in a network become deteriorated. Thus, the other performance index is the degree of reduction of OD travel demand from the normal network state, that is, the ratio of the OD travel demand of a state $\mathbf{x}$ to that of the normal state $\mathbf{x}_0$. This is denoted by $q_{rs}(\mathbf{x})/q_{rs}(\mathbf{x}_0)$.

These performance indexes are calculated for each OD pair. Aggregating these indexes, performance indexes of the whole network are derived. For example, the reduction rate of total travel demand $\sum_{r,s} q_{rs}(\mathbf{x})/\sum_{r,s} q_{rs}(\mathbf{x}_0)$ can be used as a performance index. The consumer's surplus has been used as one of the appropriate evaluation measures of transport systems. That is written as,

$$CS = \sum_{r,s} \int_0^{q_{rs}} D_{rs}^{-1}(y)dy - \sum_{r,s} q_{rs}u_{rs} \tag{6}$$

Similar to the other performance indexes, the ratio of CS at state $\mathbf{x}$ to the normal state $\mathbf{x}_0$, $CS(\mathbf{x})/CS(\mathbf{x}_0)$ is used as an index. When the travel demand function is separable, we can evaluate the consumer's surplus at each OD pair.

3.2 Definition of Reliability

We will define the reliability of a flow network using a performance index. A reliable transport network generally means the network in which one can travel from his/her origin to the destination without much uncertainity. The state of a network is probablistic and the performance indexes are also random variables. Therefore, we define the reliability as the probability of whether a performance index is sustained within an acceptable level. The probability is written as:

$$R(c)=\begin{cases} \text{Prob.}[PI(\mathbf{x}) \leq c] \text{, when } PI(x) \text{ is increasing.} \\ \text{Prob.}[PI(\mathbf{x}) \geq c] \text{, when } PI(x) \text{ is decreasing.} \end{cases} \tag{7}$$

where $PI(\mathbf{x})$ is the value of a performance index at state $\mathbf{x}$. Parameter c denotes an acceptable level. The value of c is exogenously determined considering the level of service that should be maintained even in deteriorated situations. When a performance index is for each OD pair, R(c) is defined as $R_{rs}(c)$.

Equation (7) is a general definition of network reliability. When the ratio of OD travel time, $u_{rs}(\mathbf{x})/u_{rs}(\mathbf{x}_0)$, is used as a performance index, $R_{rs}(c)$ becomes the travel time reliability. It means the probability of whether one can travel from origin r to destination s within an acceptable delay rate c. When the value of c is given as infinite, $R_{rs}(c)$ is consistent with the connectivity measure discussed in the previous studies on system reliability.

The connectivity measure in the previous reliability studies implicitly assumed that network users would divert to any alternative route. Even if the OD travel time along the diverted route is remarkably large, users would use the route. For the same OD pair, the network covering a wider area becomes more reliable than that covering a smaller area. The nearly infinite diversion will be applied to a communication network, however it may not always be suitable for evaluating transport networks.

3.3 Approximation Algorithm

The value of R(c) is estimated using the expected value of the operated/failure function, which determines whether the performance index is within the given level. Taking an increasing performance index such as OD travel time ratio, the oprated/failure function is written as,

$$Z(c,\mathbf{x})=\begin{cases} 1 & \text{if} \quad PI(\mathbf{x}) \leq c \\ 0 & \text{if} \quad PI(\mathbf{x}) > c \end{cases} \tag{8}$$

Note that the subscripts r-s are dropped from $Z_{rs}(c,\mathbf{x})$, $PI_{rs}(\mathbf{x})$ and $R_{rs}(c)$ in order to avoid complexity. The probability R(c) is the mathematical expectation of $Z(c,\mathbf{x})$ weighted by the state probability $p(\mathbf{x})$, which is written as:

$$R(c) = E[Z(c,\mathbf{x})] = \sum_{\mathbf{x} \in X} p(\mathbf{x})\, Z(c,\mathbf{x}) \tag{9}$$

When the number of all state vectors is not so large, Eq.(9) can be directly calculated. However, the number of state vectors is generally very large and the direct calculation of Eq.(9) becomes difficult. For a network with L links, the number of possible states amounts to 2^L. If the UE traffic assignment is calculated for each network state, the direct calculation requires huge computation cost. Thus, we show an algorithm for approximating R(c). The original idea was presented by Li and Silvester (1984). The algorithm defines the lower and upper bounds using the J most probable state vectors.

Sorting state vectors in the order of the state probability as Eq.(10):

$$p(\mathbf{x}_1) \geq \ldots \geq p(\mathbf{x}_j) \geq p(\mathbf{x}_{j+1}) \geq \ldots \geq p(\mathbf{x}_N) \tag{10}$$

where $\mathbf{x}_j$ denotes the j-th most probable state vector, $p(\mathbf{x}_j)$ represents the state probability for the state $\mathbf{x}_j$ and N is the number of all state vectors. Using the state vectors by the J-th most probable state vector $\{\mathbf{x}_1,\ldots,\mathbf{x}_J\}$ and corresponding the values of the operated/failure function $\{Z_{rs}(c,\mathbf{x}_1), \ldots, Z_{rs}(c,\mathbf{x}_J)\}$, the upper and lower bound of the expected value can be defined as follows.

$$R^U(c) = \sum_{j=1}^{J} p(\mathbf{x}_j)\, Z(c,\mathbf{x}_j) + \left(1 - \sum_{j=1}^{J} p(\mathbf{x}_j)\right) \tag{11.a}$$

$$R^L(c) = \sum_{j=1}^{J} p(\mathbf{x}_j)\, Z(c,\mathbf{x}_j) \tag{11.b}$$

The expected value of R(c) stays between $R^U(c)$ and $R^L(c)$. The upper bound and the lower bound converge to the exact expected value of R(c) from the upper side and the lower side, respectively. We take the next most probable state vectors one after another and update the approximated expected value of the reliability measure R(c) until the difference between the upper and lower bounds becomes small enough. Figure 3 shows an image of convergence of the approximation algorithm.

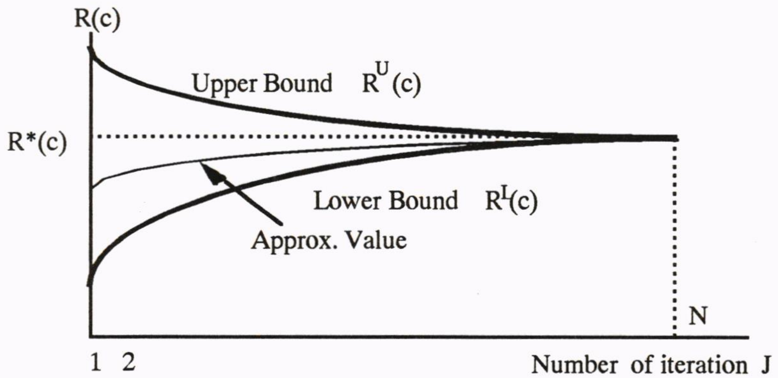

Figure 3 An Image of Convergence of Approximation Algorithm

The algorithm is summarized as follows.

Step.0: *Normal state.* Calculate the network flows by solving the UE problem for the normal state $\mathbf{x}_0$. Set iteration counter J=1.

Step.1: *J-th probable state.* Take the J-th most probable state vector $\mathbf{x}_J$, where the probability $p(\mathbf{x}_J)$ is the J-th largest.

Step.2: *Network flow.* Calculate the network flows by solving the UE problem for the state $\mathbf{x}_J$.

Step.3: *Performance indexes.* Compare the network flows at state $\mathbf{x}_J$ with those at normal state $\mathbf{x}_0$ and obtain the value of performance indexes. Then, evaluate the operated/failure function $Z(c,\mathbf{x}_J)$ for each performance index.

Step.4: *Upper and lower bounds.* Calculate the upper and lower bounds using Eq.(11.a) and Eq.(11.b), respectively.

Step.5: *Convergence check.* If the difference of the upper and the lower bounds is small enough, go to Step.6. Otherwise, set J=J+1 and return to Step.1.

Step.6: *Approximation.* The expected value of $R_{rs}(c)$ is approximated as,

$$R(c) = \{R^U(c) + R^L(c)\}/2 \tag{12}$$

4. NUMERICAL EXAMPLES

4.1 Input Data

Small scale network examples are calculated to compare some reliability models using different performance indexes. Figure 1 shows a test network with 9 nodes and 12 dual directed links. Links are categorized into three link types; primal distributer, local distributer and access road. Table 1 shows the free flow travel time and capacity of those links. The Davidson function of Eq.(3) is used as link cost function and the exponential function of Eq.(4) is applied for demand function between each OD pair. The parameters of those functions, κ and γ, are given as 0.5 and 0.02, respectively. All nodes in the network are assumed to generate and attract traffic. The amount of potential demand D^0_{rs} is given as 1,000 for each OD pair.

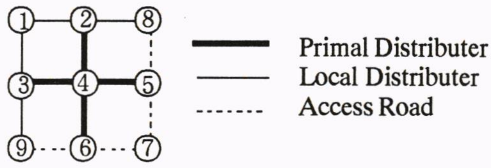

Figure 4 Test Network

Table 1 Types and Attributes of Links in the Test Network (non-dimension)

Link Types	Free Flow Travel Time t_{a0}	Link Capacity C_a
Type 1: Primal Distributer	2	15,000
Type 2: Local Distributer	3	10,000
Type 3: Access Road	4	5,000

Link connectivity $p_a(F)$ is defined as the probability of whether link a is physically connected and maintaines its function when the external force F is loaded onto the link. If the magnitude is moderate, the link is expected not to be damaged at all. The probability monotonically decreases when the external force becomes larger. The degree of decrease is assumed to be different among link types. Among the three link types, a primal distributer keeps the highest connectivity, a local distributer the second and an access road the third. Figure 5 depicts the relations of the link connectivity and the magnitude of the external force and their differences among link types. For example, a link categorized as local distributer is assumed connected if the magnitude of the external force is less than 1.5, and then the connectivity of the link decreases linearly until the magnitude reaches 11.5.

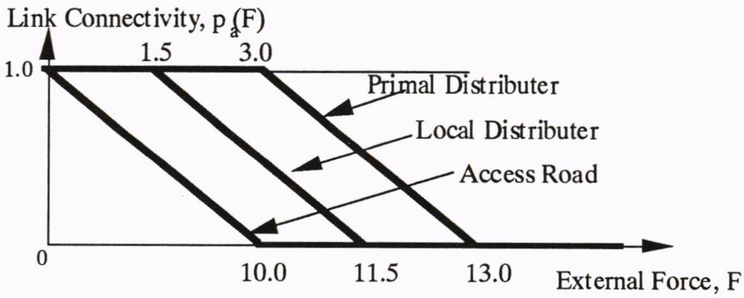

Figure 5 Link Connectivity $p_a(F)$ for External Force F

4.2 Computational Results

Figure 6 depicts the values of travel time reliability, $\text{Prob.}[u_{rs}(\mathbf{x}) / u_{rs}(\mathbf{x}_0) \le c]$, between OD pair 1-7 and OD pair 8-9 for different criterion c. The magnitude of external force is fixed at 2.0, in which the values of connectivity of three link types are 1.0 for primal distributer, 0.95 for local distributer, and 0.80 for access road, respectively. When the value of c is small, the OD pair is liable to be judged unconnected. When the criterion c becomes larger, a longer detour is acceptable and the reliability of the OD pair becomes higher. When the criterion is extremely large, the time reliability of an OD pair in a flow network yields to the connectivity in a pure network. For almost all of the values of c, the OD pair 1-7 is less reliable than the OD pair 8-9, because all links connecting with node 7 are access roads with a lower value of link connectivity.

Figure 7 shows the values of travel demand reliability, $\text{Prob.}[q_{rs}(\mathbf{x}) / q_{rs}(\mathbf{x}_0) \ge c]$, between OD pair 1-7 and OD pair 8-9 for different criterion c. The magnitude of external force is also fixed at 2.0. The reliability is decreasing for criterion c, because an OD pair is judged to be connected for a lower value of c only if a small amount of OD demand exists. When the value of c approaches zero, the reliability converges to the connectivity in a pure network.

Figure 8 represents the value of travel time reliability between OD pair 1-7 and OD pair 8-9 for different magnitudes of external force. The criterion c is fixed at 1.5, which means 1.5 times of detour time in OD travel time is judged acceptable. In proportion to the magnitude of the external force, the values of time reliability are decreasing. The OD pair 8-9 is very reliable when the magnitude is less than 1.5. The OD pair 1-7 is less reliable and the reliability value for the highest magnitude is almost half of that for the lowest magnitude.

Figure 9 depicts the fluctuation of the ratio of the consumer's surplus in the whole network against the magnitude of the external force. The ratio is written as $E[CS(\mathbf{x})]/CS(\mathbf{x}_0)$, where $E[CS(\mathbf{x})]$ denotes the expected value of the consumer's surplus of a network as a whole $E[CS(\mathbf{x})] = \sum_{\mathbf{x} \in X} p(\mathbf{x}) CS(\mathbf{x})$ and $CS(\mathbf{x}_0)$ is the consumer's surplus of a network for the normal state. The ratio is culculated for any magnitude of external force. It is found that the ratio decreases monotonically as the external force increases. This implies the performance measured by the consumer's surplus of a network as a whole yields worse when the magnitude of the external force becomes larger.

Figure 10 shows the values of the reliability defined using consumer's surplus $\text{Prob.}[CS(\mathbf{x}) / CS(\mathbf{x}_0) \ge c]$ for different degrees of the external magnitude. When the criterion c is set 0.8, the probability remains more than 0.8 at the highest magnitude of the external force. This means that the network is judged reliable if a 20% reduction of the

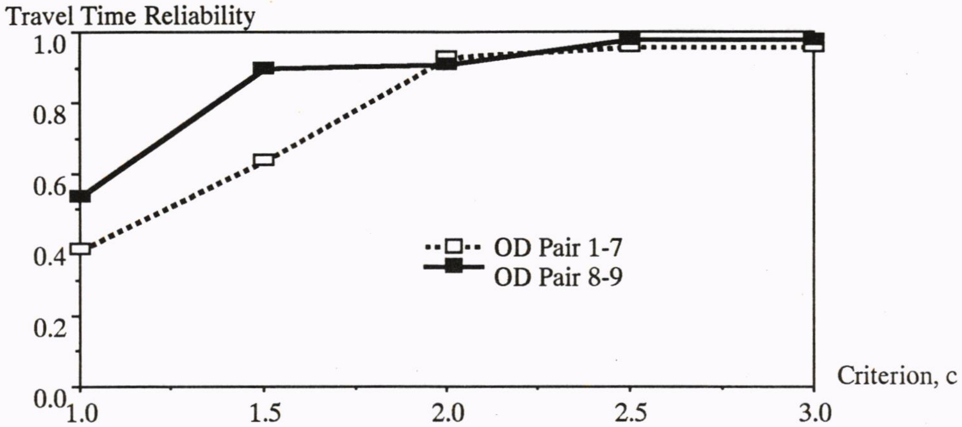

Figure 6 Travel Time Reliability between OD Pairs 1-7 and 8-9 (F=2.0).

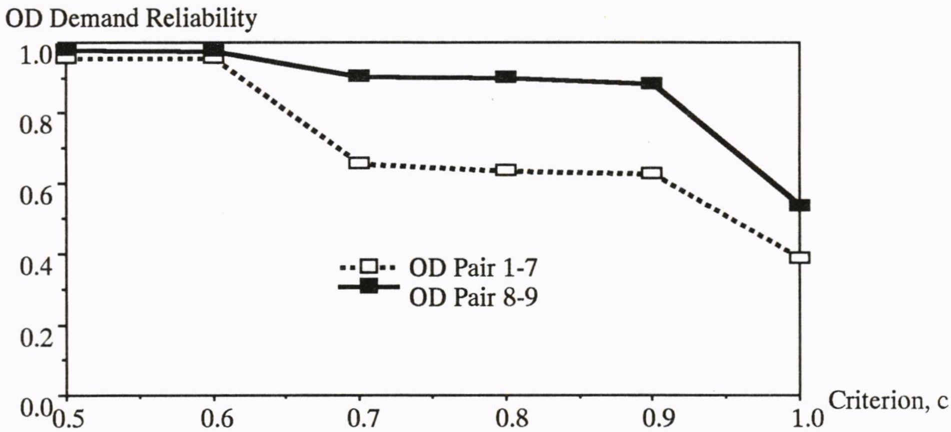

Figure 7 OD Demand Reliability between OD Pairs 1-7 and 8-9 (F=2.0)

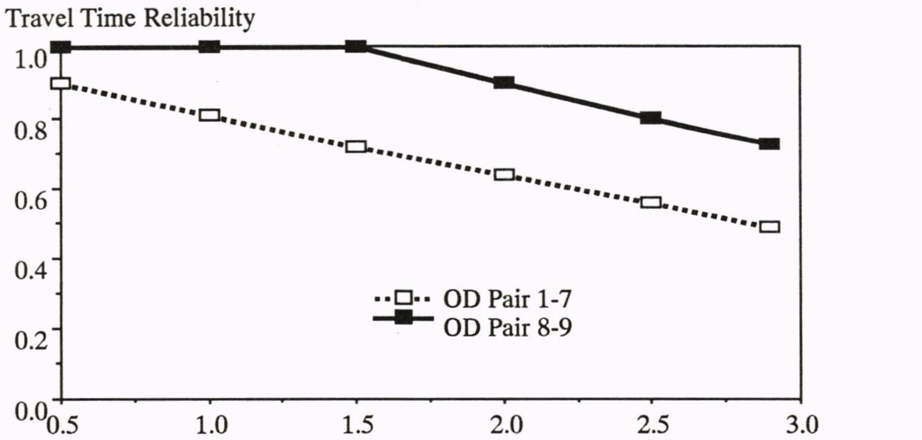

Figure 8 Travel Tme Reliability for Different Magnitude of F

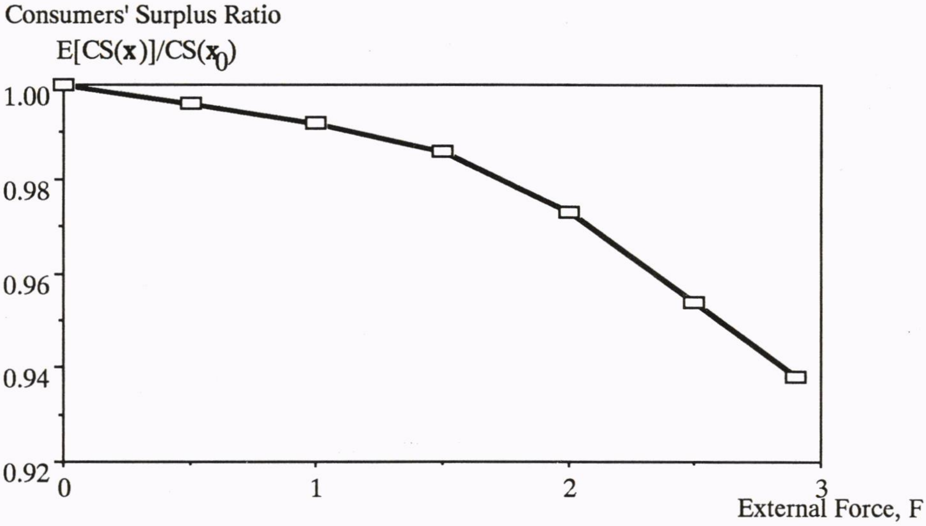

Figure 9 Expected Value of Consumer's Surplus of Whole Network

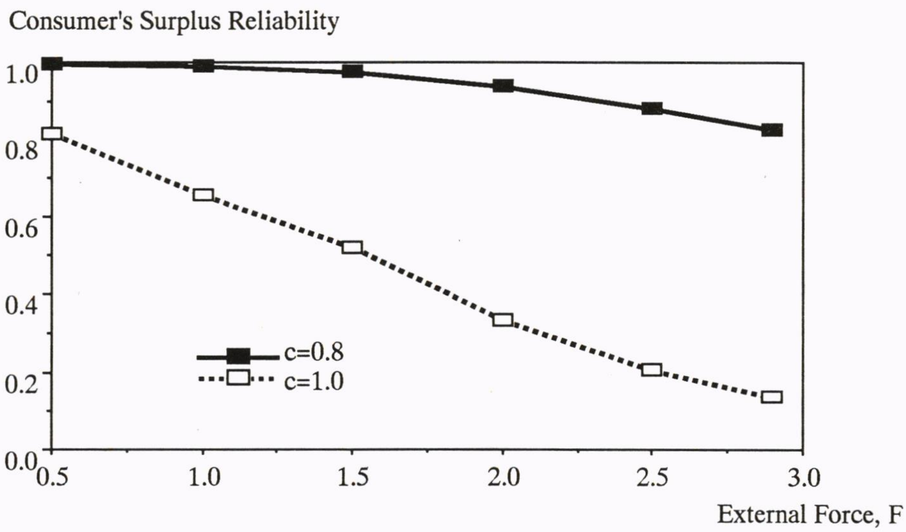

Figure 10 Consumer's Surplus Reliability of Whole Network

consumer's surplus is acceptable. However, the network is judged unreliable when the criterion c is set 1.0 implying the consumer's surplus must be kept at the level for the normal state.

5.CONCLUSION

The state of a network is not deterministic since some links may be physically damaged by disasters and closed to traffic. This paper proposes some models for evaluating the reliability of a network deteriorated by natural disasters. Reliability is defined as the probability of whether the performance of a network or an OD pair is kept within an acceptable level. The reliability indexes are shown using the travel time and travel demand between an OD pair, and the consumer's surplus of a network as a whole.

In order to reduce computational cost for estimating the reliability measure, an approximation algorithm is presented. Through numerical examples, it is found that the proposed reliability models could be available for evaluating the performance of a deteriorated network.

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