THE MINIMUM HEADWAY OF A RAIL TRANSIT LINE

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abstract: The paper reports the testing results of a simulation study on the estimation of minimum headway for the orange line of the Kaohsiung Mass Rapid Transit (KMRT). The simulation results of minimum headway are further compared with the values of minimum headway calculated by the theoretical formula which have been widely discussed in railway literature.

1. INTRODUCTION

The minimum headway is an important factor for the planning, design , and operation of a rail transit line (Nigel et al., 199). The calculations of minimum headway are discussed in detail in many books of railway or public transportation (Vuchic, 1981; Nock, 1993). System simulation is a widely used technique in railway planning and operation (Yoshikawa, 1992) , and it is used to generate realistic estimates of the minimum headway in this study. The results of a simulation study on the effect of headway for the orange line of Kaohsiung Mass Rapid Transit (KMRT) are presented in the paper, and they are further compared with the values of minimum headway calculated on the base of the formula described in the railway literature.

2. OVERVIEW ON THE THEORY OF MINIMUM HEADWAY

The formula of the minimum headway in railway traffic flow theory and that in railway signaling theory are widely discussed (Vuchic, 1981; Nock, 1993). They are briefly reviewed in this section.

2.1. Railway Traffic Flow Theory

If a train may stop instantaneously, the separation between two successively moving trains must be larger than or equal to the braking distance of the second train. Having a steady speed 'v' and a minimum braking rate 'b', the minimum separation or braking distance is given by (1).

$$s = \frac{v^2}{2b} \qquad (1)$$

If the train length is 'l', then the minimum headway 'H' is written as (2).

$$H = \frac{s+l}{v} \qquad (2)$$

To minimize 'H', the optimum running speed 'V' is given by (3).

$$V = \sqrt{2bl} \qquad (3)$$

At last, take (3) into (2), the minimum headway is rewritten as (4).

$$H = \sqrt{\frac{2 \cdot l}{b}} \qquad (4)$$

2.2. Railway Signaling Theory

Fixed block signaling is the most widely used form of signaling, both for urban and inter-city railway operations. Consider a 3-aspect arrangement as shown in Figure 1, the minimum headway distance 'h_3 'is given by (5).

$$h_3 = 2\,d_3 + p + o + l \qquad (5)$$

where 'd_3' is the block length for 3-aspects,

'p' is the sight distance,

'o' is the overlap distance beyond the signal, and

'l' is the length of train length.

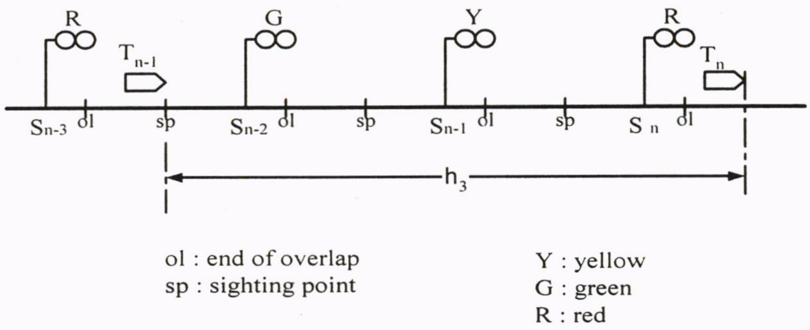

Figure 1: A 3-Aspect Fixed Block Layout

In practice, the 3-aspect block length is the braking distance 's'. Furthermore, if the 4-aspect is incorporated as shown in Figure 2, the minimum headway distance 'h_4 ' is then written as (6).

$$h_4 = 3\,d_4 + p + o + l \qquad (6)$$

The standard practice to the 4-aspect block length is one half of braking distance's'. It follows that the minimum headway distance for a n-aspect arrangement is given by (7).

$$h_n = \frac{n-1}{n-2} s + p + o + l \qquad (7)$$

In order to minimize the headway distance, the optimum speed is given by (8).

$$V = \sqrt{2bk} \tag{8}$$

where $k = p + o + l$

Therefore, the minimum headway at the optimum speed for n-aspect can be rewritten as (9).

$$H = \sqrt{\frac{2k(n-1)}{b(n-2)}} \tag{9}$$

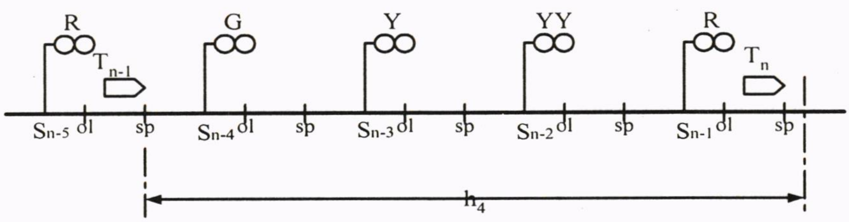

Figure 2: A 4-Aspect Fixed Block Layout

2.3. Discussion

When 'n' is a very large number or the moving block signaling is selected, and/or if an intelligent system is chosen to decrease the value of 'p' and 'o', the limit of the minimum headway in formula (9) approaches to that in formula (4). In the minimum headway formula (4), the headway is only dependent on the train length and the braking rate. It implies that a short train will result in a short headway. However, in the design and operation of a transit line, we have to consider not only the headway but also the capacity.

A lot of design and operation factors have not been considered in the formula of minimum headway mentioned above. Examples are the geometric factors of the railway line, such as curves and gradients, the mechanical characteristics of the vehicle, such as the traction and acceleration capabilities, and the practical factors in operation, such as the number of stops and the platform dwell time at a station. Therefore, the minimum headway calculated by the formula may be quite different from the operational minimum headway in practice.

3. OVERVIEW OF A SIMULATION MODEL

In order to find the realistic minimum headway for a specific railway, a simulation model is developed. It mainly consists of two parts: a train performance simulator and a train movement simulator (Lee et al., 1997). They are respectively described briefly in the following.

3.1. The Train Performance Simulator

The heart of the simulation model is a train performance simulator. It simulates the motion of a train along a railway so as to obtain the trajectory of the train, given the input data which represents the track configurations, the vehicle operating characteristics, and operation conditions. That is, the train performance simulator can generate the output which describes the speed, time and traction of the train over distance.

It is well known that the speed and running time of a train can be calculated by solving the differential equations derived from Newton's law of motion (Inada et al., 1975; Andrews, 1986). However, it is much easier and more efficient to obtain the trajectory of a train by discrete simulation techniques (Uher et al., 1987). Assuming the acceleration of the train is a constant over a very short section, the equations of motion can be integrated to compute the speed of the train at the end of the section with a given speed at the beginning of the section. Similarly, given the train speed at the end of the section, the speed of the train at the beginning of the section can be calculated in the direction of negative time flow. An example of the forward and backward calculations is illustrated in Figure 3. In summary, the train performance simulator represents the functions of system units instead of simulates the operations of the units. The train performance simulator is developed to plot the trajectory of a train movement in terms of velocity-distance, acceleration-distance, and so on.. However, there is no consideration of the interactions among the trains on the railway network.

3.2. The Train Movement Simulator

The primary objective of the train movement simulator is to simulate the commands from the railway traffic control system to each train on the network. In general, the traffic control system consists of a fixed block signaling system and a centralized dispatching system. According to the traffic conditions at each simulation time step, the train movement simulator updates the display of each block signal and generates commands from the dispatcher and/or to the drivers, which then result in new constrains for calculating the trajectory of train movements at and after the time step. An example of the effect of the block signal on the calculation of the trajectory of two trains is illustrated in Figure 4. Note that the backward speed constrains due to the signals are dependent on the movement of the preceding train, and these constrains have effect on the movement of the successive train.

With the consideration of the traffic control system in the train movement simulator, the model can simulate the relationships of all trains on the railway network at each discrete and adjustable time step, where the movement of each train at each time step is calculated by the train performance simulator. In summary, the train movement simulator is a toll to represent the traffic control system of the railway network, and it takes care of the interactions among the movements of trains.

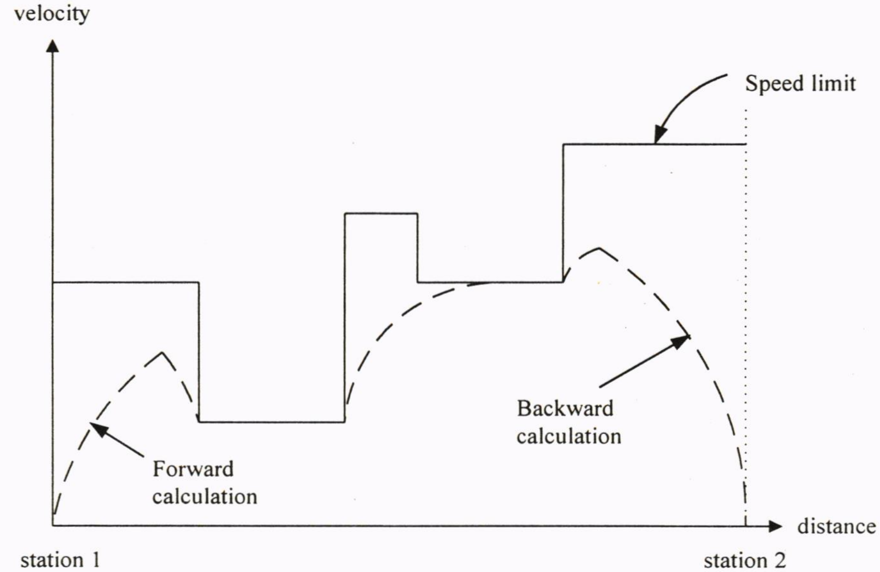

Figure 3: Trajectory Calculation

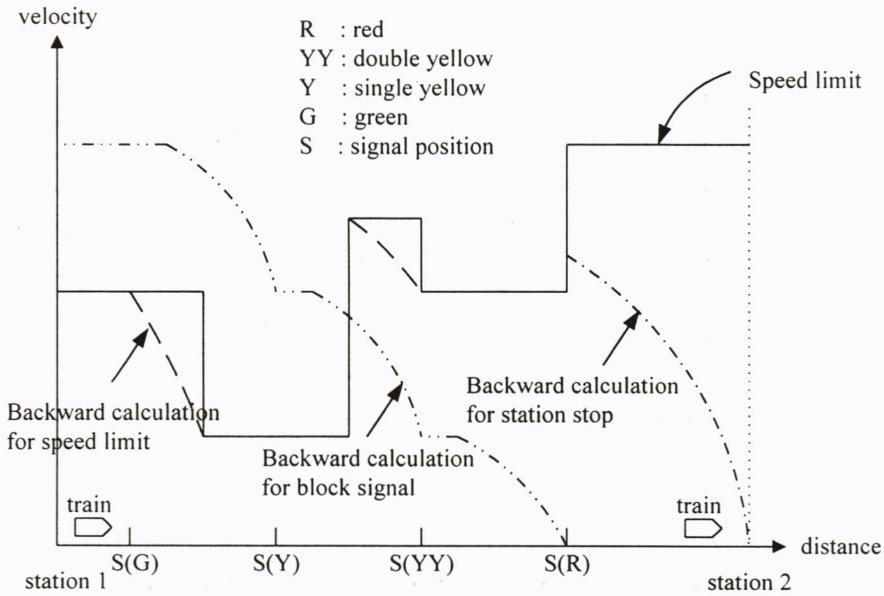

Figure 4: The Effect of Block Signal on Trajectory Calculation

4. THE MINIMUM HEADWAY OF THE ORANGE LINE

The Kaohsiung Mass Rapid Transit (KMRT) is in the planning and design stage. The KMRT orange line is one of the proposed projects which have been studied intensively (International Transit Consultants, 1993) . The orange line is located from the east end of Kaosiung to the west, its total length is 14 kilometers, and it consists of 15 stop stations. According to a demand analysis, the required minimum headway for the orange line is 200 seconds. Moreover, the orange line's basic track configurations, vehicle and traction characteristics, and types of traffic control systems have also been briefly studied. The data of the proposed track configurations- such as the speed limit at each section of the line, the proposed vehicle characteristics- such as the train length and traction capability, and an assumed 4-aspect block signaling system are used in the simulation study on the estimation of minimum headway for the orange line. The simulation model described in the previous section was developed in FORTRAN and all experiments were run on a personal computer 586.

4.1. Testing Results

Headway is in general considered as the time between successive trains such that the speed of the following train is not restricted by the position of the preceding train. After many simulation runs, the minimum headway of the orange line is obtained as 150 seconds. As the time-space diagram shown in Figure 5, every train meets the green light at every signal position from the first station to the end terminal.

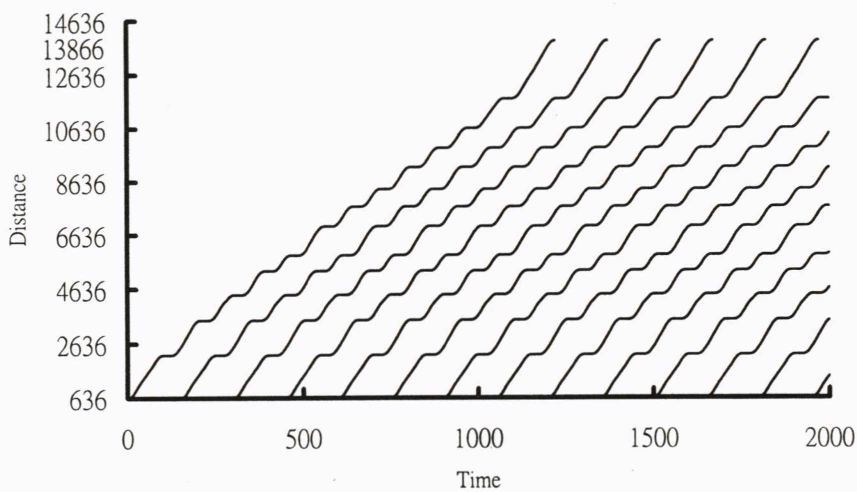

Figure 5: The Time-Space Diagram for 150-second Headway

If a train is permitted to meet other lights instead of green, the time interval between successive train can be shorter than 150 seconds. For example, the time-space diagram of 80-second headway is illustrated in Figure 6. It is clear that the interruption between successive trains happened. The total running time for one of the last few trains is longer than the normal total running time for a train that meets only green light.

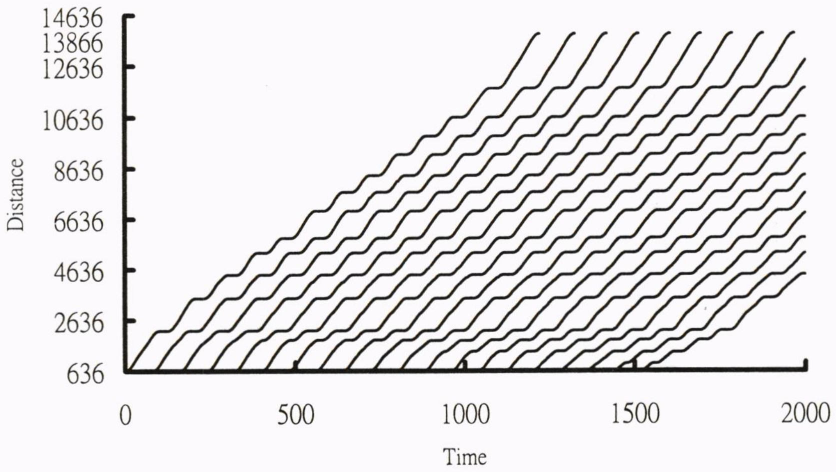

Figure 6: The Time-Space Diagram for 80-second Headway

Delay is defined as the difference between the actual running time and the normal running time without interruption. There is no delay for every train if the headway is greater than or equal to 150 seconds. As the results shown in Figure 7, the average delay or the delay of the last train in the peak hour is increasing slowly, when the headway is shorter than 150 seconds but greater than 90 seconds. Therefore, the operational headway may be shorter than the design minimum headway- 150 seconds, if the safety of the system is still guaranteed and the amount of delay is acceptable. However, as also shown in Figure 7, the delay is increasing very fast when the headway is lesser than 90 seconds. Therefore, the cost for an decrease of headway, when it is less than 90 seconds, is quite high, with regard to train delay.

The capacity can be measured by the train-kilometers run in the period of interest, because the train length is generally fixed during peak hours. Then, as the simulation results illustrated in Figure 8, the capacity will in general increase as the headway is decreased. However, the capacity is saturated when the headway approaches to 90 seconds. Hence, with the consideration of delay and capacity, there is no reason to operate the system at the headway lesser than 90 seconds.

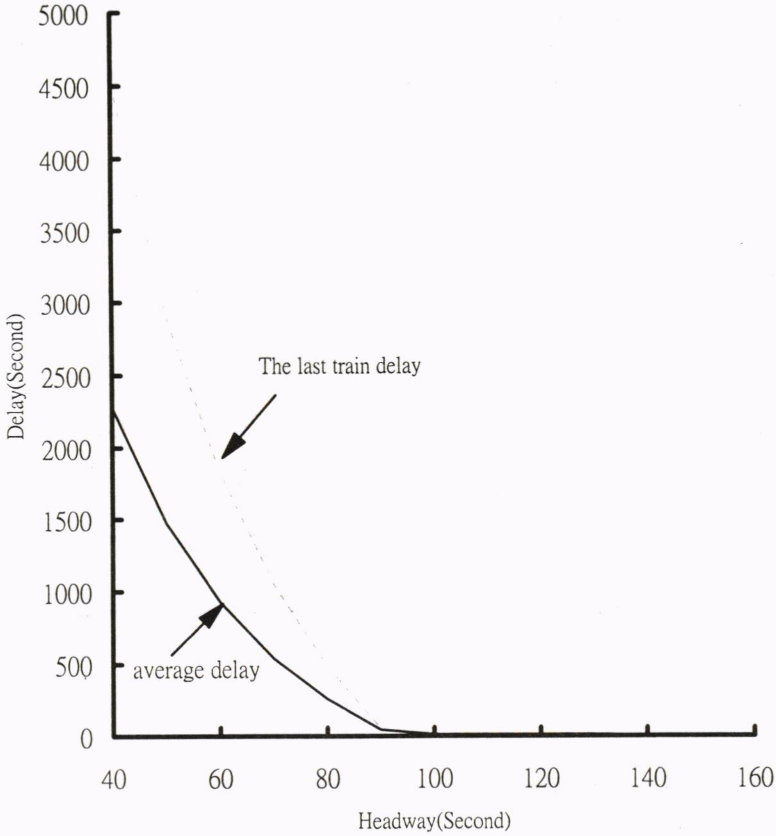

Figure 7: The Relationship between Delay and Headway

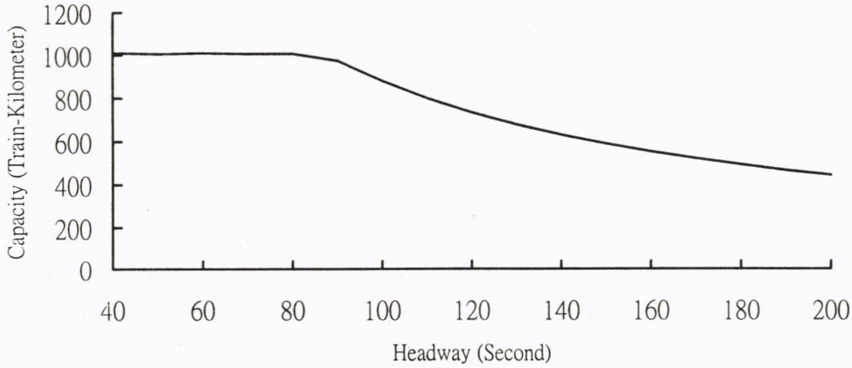

Figure 8: The Relationship between Capacity and Headway

4.2. Comparison between the Theoretical and Simulated Minimum Headway

As mentioned above, the simulated minimum headway for the KMRT orange line is 150 seconds. However, the minimum headway based on the railway traffic theory or formula (4) is 17 seconds, because the train length is 132 meters and the braking rate is 1 meter per second square. Moreover, the minimum headway based on the railway signaling theory or formula (9) is 42 seconds, because the block distance is 250 meters, the overlap distance is 25 meters, the train length is 132 meters, the braking rate is 1 meter per second square, and a 4-aspect block signal is considered. It is clear that the difference between these values of minimum headway is quite big. Therefore, we have to use the textbook formula of minimum headway very carefully in practice. Furthermore, it is useful to obtain the simulation results of realistic estimates of the minimum headway.

In order to test the effect of the assumptions used in developing the theoretical formula of minimum headway, several simulation experiments has been done. First, assume that there is no speed limit, then, the minimum headway is obtained as 140 seconds. That is, the curves and gradients on the orange line have only 10-second effect on the minimum headway. Secondly, assume that there is no speed limit and there is no platform dwell time at each station; then, the minimum headway is obtained as 100 seconds. It seems that the 25-second dwell time at each station is an important factor for the decrease of headway. Therefore, it is essential to keep the dwell time short in practical operation, so that the headway can be kept short. Thirdly, assume that there is no speed limit, no dwell time at each station, and no stop along the whole line; then, the minimum headway is obtained as 55 seconds. The 45-second difference represents the effect of acceleration and deceleration of the train for its midway stops. Hence, the effect of train stops is quite obvious. This result may also indicate the capability of the French ARAMIS system. In ARAMIS, vehicles can separate and/or connect to the train automatically when it is close to a station. Thus, the train does not have to stop at any midway station. Moreover, if the time used in the acceleration at the initial station and that used in the deceleration at the terminal station are deducted from 55 seconds, the result will be close to 42 seconds, which is obtained on the base of railway signaling theory or formula (9). Finally, the 25-second difference between the 42 seconds based on railway signaling theory and the 17 seconds based on the railway traffic flow theory may represent the maximum possible improvement for intelligent traffic control system on the minimum headway.

4.3. The Stochastic Effect of Platform Dwell Time

The running time of a train on a modern urban rail transit system is very accurate because of its advanced automation equipment. But the platform dwell time is in general not a constant because it is partly affected by the uncertainty of passenger volume. In order to investigate its stochastic effect, .the platform dwell time was tested in some simulation experiments as a stochastic variable 'w'. Because the orange line has no operation data to estimate the function of 'w'. It is simply defined as (10):

$$w = 25(1 \pm r) \qquad (10)$$

where 25 is the standard dwell time, and

r is a random number uniformly between 0 and 5%, 10%, or 20%.

As shown in Table 1, the mean of the last train delay of the peak hour with stochastic dwell time is in general longer than that with 25 seconds dwell time. The difference illustrated in table 1 is big if the confidence interval includes 0 (Law, 1991) . However, for the three tested values of headway, their difference is significantly different from 0 only if headway equals to 80 seconds. Moreover, because the platform dwell time is in general longer than the standard 25 seconds during the peak hour, 'w' is redefined as (11):

$$w = 25(1 + r) \qquad (11)$$

As also shown in Table 1, the mean of the train delay with the stochastic dwell time of (11) is in general longer than that with the stochastic dwell time of (10) and that with 25 seconds dwell time. Furthermore, the difference between the train delay with the stochastic dwell time of (11) and that with 25 seconds dwell time is significant different from 0 no matter what value of headway is tested. Therefore, the stochastic effect of platform dwell is an important factor to find the operational minimum headway in practice.

Table 1 : The Stochastic Effect of Platform Dwell Time on Train Delay

The last train delay (second)	Headway =150	Headway =120	Headway =80
25-second dwell time	0 second	13 seconds	487 seconds
25(1±5%)	0+0.87 〔-0.4,2.2〕	13+1.46 〔-0.3,3.2〕	487+5.71 〔1.3,10.1〕
25(1±10%)	0+0.72 〔-1.7,3.1〕	13+3.06 〔-1.3,7.4〕	487+10.8 〔3.0,17.3〕
25(1±20%)	0+0.95 〔-3.8,5.7〕	13+4.97 〔-5.0,8.9〕	487+21.66 〔9.5,12.2〕
25(1+5%)	0+4.54 〔3.8,5.3〕	13+5.06 〔3.7,6.5〕	487+22.80 〔19.2,26.4〕
25(1+10%)	0+8.45 〔6.9,10.0〕	13+9.22 〔6.6,11.9〕	487+41.22 〔36.7,47.1〕
25(1+20%)	0+17.15 〔14.1,20.2〕	13+17.10 〔12.3,21.9〕	487+82.36 〔72.0,92.4〕

0+0.87 represents 0+the mean of the difference; the sample size = 10;

〔Confidence Interval〕.

5. CONCLUDING REMARKS

The paper reports the simulation results of the minimum headway for KMRT orange line. They are quite different from the values calculated by the theoretical formula discussed in railway literature. However, the differences can be well explained in accordance with the assumptions used in the development of the formula. Moreover, if the interruption between successive trains is permitted and some delay of the trains is acceptable, the operational headway can be shorter than the designed minimum headway, where the following train is not restricted by the proceeding train.

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