TESTING OF A TRAIN DISPATCHING MODEL

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abstract: The paper presents a mixed integer programming model of the train dispatching problem. The model can be used to study the optimal timetable and train digram for a realistic railway system. Moreover, it is shown to be a useful experimental tool to investigate the operational characteristics of any train service plan.

1. INTRODUCTION

The elements of a railway traffic planning system include: calculation of standard runnings, preparation of timetable and train digram, engine and vehicle rostering, crew scheduling, station and yard work scheduling, and so on [Iida, 1975; Hirose et al., 1994]. In order to construct an efficient timetable and train digram, one of the most difficult problems-the train dispatching problem has to be solved effectively [Genser, 1977]. The problem is stated as follows: given a rail line, two opposing fleets of trains traversing that line, and an ideal timetable of scheduled arrival and departure times at scheduled stop stations for each train, it is to find the operation plan to move the trains from their origins to final destinations that is consistent with physical and operational constraints so as to minimize the total delay [Jovanovic, 1989]. The primary challenge in solving such a problem is to determine the appropriate locations for each conflicting train to overtake and/or to meet another train.

For Taiwan Railway Administration (TRA), the train dispatching problem is very complex. First, the number of physical constraints is very large because the railroad is mixed with three types of rail track sections, several types of station layouts, and a few types of traffic control systems. Secondly, the number of operational constraints is also very large because there are more than ten types of train services, seven classes of train speeds, and more than 1900 train operations per day. Recently, TRA is developing a computer-aided expert system to prepare the timetable and train digram. In this paper, mathematical programming techniques will be used to investigate the characteristics of the train dispatching problem for such a complex railway system.

In the next section, after a brief review on the existing models of optimal train dispatching, a mixed interger programming model will be presented. It follows that the model will be verified with small examples. Then, some testing results of the model will be shown to depict the operational characteristics of mixed train services.

2. MODEL BUILDING

Many heuristic algorithms and interactive computer-aided systems have been developed for the train dispatching problem of practical size[Szpigel, 1973; Hwang, 1977; Sauder et al., 1986; Heber et al., 1992; Kataoka, 1994; and Cai et al., 1994]. However, there is not much literature in mathematical programming on the formulation and solution procedure for the train dispatching problem. One reason is that it is difficult to write the physical and operational constraints of a complex train dispatching problem accurately in a mathematical programming model [Perterson et al., 1986; Jovanovic, 1989]. The other reason is that the train dispatching problem is a NP-hard problem to be solved exactly [Cai et al., 1994]. Nevertheless, in order to understand the characteristics of the train dispatching problem, it is worthy to build a rigorous model with a clear, accurate and closed form.

2.1. Notation

Let m = the index of a station, where $1 \leq m \leq N$;
I = the set of outbound trains, $i \in I$; and
J = the set of inbound trains, $j \in J$.

For each outbound train $i, i \in I$; the index :

σ_i = the origin station of outbound train i;
ε_i = the destination station of outbound train i;
O_i = the set of meetpoints or stations for outbound train i; and
S_i = the set of indices of scheduled stop stations for outbound train i.

Moreover, for each outbound train $i, i \in I$; the parameter :

α_i^m = the ideal arrival time of outbound train i at scheduled stop station m;
δ_i^m = the ideal departure time of outbound train i at scheduled stop station m;
$\delta_i^m - \alpha_i^m$ = the ideal dwell time at scheduled stop station m; and
vector $\tau_i = (\tau_i^{\sigma_i}, \ldots, \tau_i^m, \tau_i^{m+1}, \ldots, \tau_i^{\varepsilon_i - 1})$, where τ_i^m = the standard running time of outbound train i from station m to station $m+1$.

Accordingly, for each inbound train j, we have $\sigma_j, \varepsilon_j, S_j, \delta_j^m, \alpha_j^m, \tau_j$.

With consideration of the traffic control system and its associated safety regulation for train operation, the parameter :

η_{ik}^m = the minimum safety headway that allows outbound train k to follow outbound train i through the station m without delay, where $i, k \in I$;

η_{jr}^m = the minimum safety headway that allows inbound train r to follow inbound train j through the station m without delay, where $j, r \in J$;

ρ_{ij}^m = the minimum safety headway at the station m between the arrival of the outbound train i and the departure of the inbound train j;

ρ_{ji}^m = the minimum safety headway at station m between the arrival of the inbound train j and the departure of the outbound train i; and

$\theta_i(\theta_j)$ = the increase of the standard running for outbound train i (inbound train j) to stop at an unscheduled station.

The decision variables, $b_{ik}^m, b_{jr}^m, c_{ij}^m, y_i^m, y_j^m$, are binary variables defined as :

$$b_{ik}^m = \begin{cases} 1, & \text{if the outbound train i departs from the station m before the outbound train k} \\ 0, & \text{otherwise} \end{cases}$$

$$b_{jr}^m = \begin{cases} 1, & \text{if the inbound train j departs from the station m before the inbound train r} \\ 0, & \text{otherwise} \end{cases}$$

$$c_{ij}^m = \begin{cases} 1, & \text{if the outbound train i traverses the segment between station m and m+1 before the inbound train j} \\ 0, & \text{otherwise} \end{cases}$$

$$y_i^m = \begin{cases} 1, & \text{if the outbount train i temporarily halt at unscheduled station m+1} \\ 0, & \text{otherwise} \end{cases}$$

$$y_j^m = \begin{cases} 1, & \text{if the inbount train j temporarily halt at unscheduled station m+1} \\ 0, & \text{otherwise} \end{cases}$$

The continuous decision variables are defined as :

a_i^m = the arrival time of outbound train *i* at station *m*;

d_i^m = the departure time of outbound train *i* at station *m*;

e_i^m = the schedule delay of outbound train *i* at scheduled station *m*, and it is a nonnegative variable;

a_j^m = the arrival time of inbound train *j* at station *m*;

d_j^m = the departure time of the inbound train *j* at station *m*; and

e_j^m = the schedule delay of inbound train *j* at scheduled station *m*; and it is a nonnegative variable.

Finnally, Ψ = a large positive number.

2.2 The Model

An optimal train dispatching model for single track rail line with the station layout shown in Figure 1(a) was built on the basis of Jovanovic's work, and it is listed as follows:

$$Min \quad Z = \sum_{i \in I} e_i^{\varepsilon_i} + \sum_{j \in J} e_j^{\varepsilon_j} \tag{1}$$

subject to :

(1) schedule constraints :

$$d_i^m = \delta_i^m + e_i^m \qquad \forall i \in I,\ m \in S_i \tag{2}$$

$$d_j^m = \delta_j^m + e_j^m \quad \forall j \in J,\ m \in S_j \tag{3}$$

(2) travel time constraints：

$$a_i^{m+1} \ge d_i^m + \tau_i^m \quad \forall i \in I, m = \sigma_i, \ldots, \varepsilon_i - 1 \tag{4}$$

$$a_j^m \ge d_j^{m+1} + \tau_j^m \quad \forall j \in J, m = \sigma_j, \ldots, \varepsilon_j - 1 \tag{5}$$

$$a_i^{m+1} \ge d_i^m + \tau_i^m + \theta_i - \psi(1 - y_i^m) \quad \forall i \in I, m = \{\sigma_i + 1, \ldots, \varepsilon_i - 1\} \setminus S_i \tag{6}$$

$$a_i^{m+1} \ge d_i^m + \tau_i^m - \psi y_i^m \quad \forall i \in I, m = \{\sigma_i + 1, \ldots, \varepsilon_i - 1\} \setminus S_i \tag{7}$$

$$a_j^{m-1} \ge d_j^m + \tau_j^m + \theta_j - \psi(1 - y_j^m) \quad \forall j \in J, m = \{\sigma_j + 1, \ldots, \varepsilon_j - 1\} \setminus S_j \tag{8}$$

$$a_j^{m-1} \ge d_j^m + \tau_j^m - \psi y_j^m \quad \forall j \in J, m = \{\sigma_j + 1, \ldots, \varepsilon_j - 1\} \setminus S_j \tag{9}$$

(3) minimum dwell time constraints:

$$d_i^m \ge a_i^m + \delta_i^m - \alpha_i^m \quad \forall i \in I,\ m \in S_i \tag{10}$$

$$d_j^m \ge a_j^m + \delta_j^m - \alpha_j^m \quad \forall j \in J,\ m \in S_j \tag{11}$$

(4) following and overtaking constraints：

$$d_k^m \ge d_i^m + \eta_{ik}^m - \psi(1 - b_{ik}^m) \quad \forall i,k \in I,\ m \in O_i \cap O_k \tag{12}$$

$$d_k^m \ge d_i^m + \eta_{ik}^m - \psi b_{ik}^m \quad \forall i,k \in I,\ m \in O_i \cap O_k \tag{13}$$

$$d_r^m \ge d_j^m + \eta_{jr}^m - \psi(1 - b_{jr}^m) \quad \forall j,r \in I,\ m \in O_j \cap O_r \tag{14}$$

$$d_r^m \ge d_j^m + \eta_{jr}^m - \psi b_{jr}^m \quad \forall j,r \in I,\ m \in O_j \cap O_r \tag{15}$$

$$a_k^{m+1} \ge a_i^{m+1} + \eta_{ik}^{m+1} - \psi(1 - b_{ik}^m) \quad \forall i,k \in I,\ m \in O_i \cap O_k \tag{16}$$

$$a_k^{m+1} \ge a_i^{m+1} + \eta_{ik}^{m+1} - \psi b_{ik}^m \quad \forall i,k \in I,\ m \in O_i \cap O_k \tag{17}$$

$$a_r^m \ge a_j^m + \eta_{jr}^m - \psi(1 - b_{jr}^m) \quad \forall j,r \in I,\ m \in O_j \cap O_r \tag{18}$$

$$a_r^m \ge a_j^m + \eta_{jr}^m - \psi b_{jr}^m \quad \forall j,r \in I,\ m \in O_j \cap O_r \tag{19}$$

$$b_{ik}^m \le b_{ik}^{m-1} \quad \forall i,k \in I, i < k,\ m \in O_i \cap O_k \tag{20}$$

$$b_{jr}^m \le b_{jr}^{m+1} \quad \forall j,r \in J, j < r,\ m \in O_j \cap O_r \tag{21}$$

(5) meeting contraints：

$$d_j^{m+1} \ge a_i^{m+1} + \rho_{ij}^{m+1} - \psi(1 - c_{ij}^m) \quad \forall i \in I, j \in I,\ m \in O_i \cap O_j \tag{22}$$

$$d_i^m \ge a_j^m + \rho_{ji}^m - \psi c_{ij}^m \quad \forall i \in I, j \in I,\ m \in O_i \cap O_j \tag{23}$$

(6) station layout constraints：

$$1 \ge (b_{ik}^{m-1} - b_{ik}^m) + (b_{kl}^{m-1} - b_{kl}^m) \quad \forall i,k,l \in I,\ i < k < l,\ m-1, m \in O_i \cap O_k \cap O_l \tag{24}$$

$$a_k^m \ge d_i^m - \psi(b_{ik}^{m-1} - b_{ik}^m) \quad \forall i,k \in I,\ i < k,\ m-1, m \in O_i \cap O_k \tag{25}$$

$$1 \ge (b_{jr}^{m-1} - b_{jr}^m) + (b_{sj}^{m-1} - b_{sj}^m) \quad \forall s,j,r \in I,\ s < j < r,\ m-1, m \in O_s \cap O_j \cap O_r \tag{26}$$

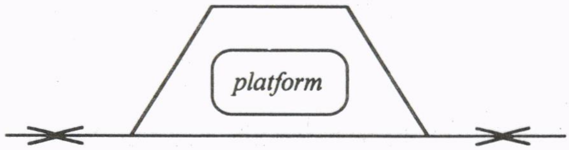
$$a_r^m \ge d_j^m - \psi(b_{jr}^{m+1} - b_{jr}^m) \quad \forall j,r \in J,\ j < r,\ m-1, m \in O_j \cap O_r \tag{27}$$

$$2 \ge (c_{ij}^{m-1} - c_{ij}^m) + (c_{kj}^{m-1} - c_{kj}^m) + (b_{ik}^{m-1} - b_{ik}^m) \quad \forall i,k \in I,\ i < k,\ \forall j \in J,\ m-1, m \in O_i \cap O_k \cap O_j \tag{28}$$

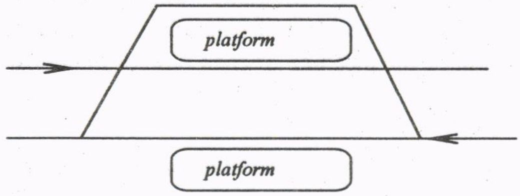
$$2 \geq (c_{ij}^{m+1} - c_{ij}^{m}) + (c_{ir}^{m+1} - c_{ir}^{m}) + (b_{jr}^{m+1} - b_{jr}^{m}) \quad \forall i \in I,\ \forall j,r \in J,\ j<r,\ m,m+1 \in O_i \cap O_j \cap O_r \tag{29}$$

$$a_k^m \geq d_i^m - \psi(2 - ((c_{ij}^{m} - c_{ij}^{m-1}) + (c_{kj}^{m} - c_{kj}^{m-1}))) \quad \forall i,k \in I,\ i<k,\ \forall j \in J,\ m-1,m \in O_i \cap O_k \cap O_j \tag{30}$$

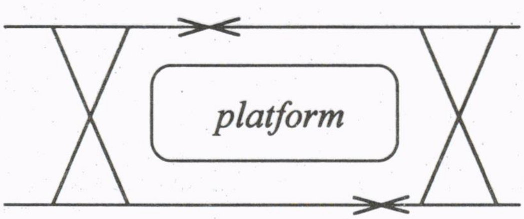
$$a_r^m \geq d_j^m - \psi(2 - ((c_{ij}^{m} - c_{ij}^{m+1}) + (c_{ir}^{m} - c_{ir}^{m+1}))) \quad \forall i \in I,\ \forall j,r \in J,\ j<r,\ m,m+1 \in O_i \cap O_j \cap O_r \tag{31}$$



(a) a station layout in sigle track rail line



(b) a station layout in double track rail line



(c) a station layout in the rail line of double track with two-way signal,

Figure 1 : Station Layouts

2.3 Discussion

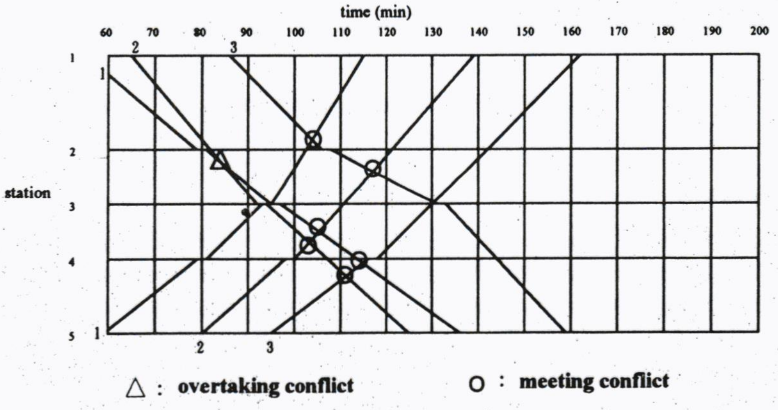
In the model, the total delay is the objective to be minimized. Every train's delay has the same weight in the objective. For practical application, the objective function can include various performance measure or measures as a generalized cost or a vector of cost functions. Moreover, in the model, the overtaking and meeting constrints are formulated only for single track rail line. There is no cross-over line and no meet point between stations. All these constraints can be modified in accordance with practical physical conditions [Lee et al., 1994]. In the next section, the testing results of three types of railroad sections- single track, double track, and double track with two-way signal will be discussed. Furthermore, the layout of sidings in a through station has a major influence on the possibilities of overtaking and/or meeting for trains. Most TRA's stations are through stations. In the model, the station layout constraints are valid only for the case shown in Figure 1(a). However, many kinds of station layouts can be formulated in the model to replace constrains (24)~(31) with practical considerations [Lee et al., 1994]. In the paper, only the testing results of the station layouts shown in Figure 1 will be presented.

3. MODEL TESTING

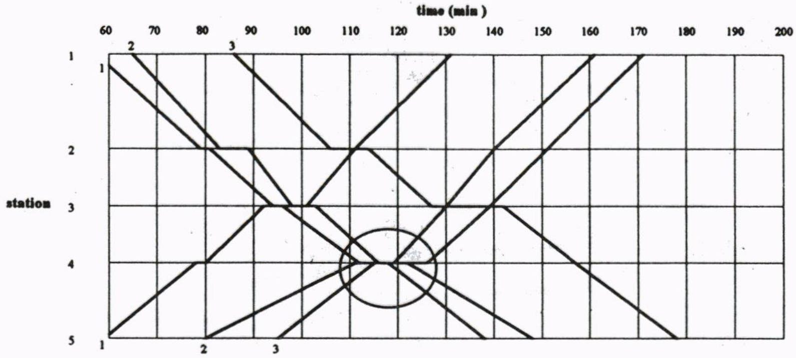
It is important to verify the validity of a model before any model implementation. For such a complicated model listed in the previous section, it is easy to have inaccurate variables and constraints. For example, some constraints may be too strict to include all feasible solutions for the problem, and some constraints may be redundant. In general, a careful examiniation on the model formulation is the first step to verify its validity. Then, the model can be tested with small examples with and without some types of constraints in order to check the meaning of each type of the constraints.

3.1 Model Verification

Many small examples were tested with the type of train dispatching model described in section 2. In this study, the branch and bound method was used to solve the model exactly by running LINDO on IBM PC [Schrage, 1991]. One example of five stations and six trains is shown in Figure 2(a). In its ideal schedule, it is clear that there are conflict points for some trains of opposing directions to meet, and conflict points for some trains of the same direction to follow or takeover. The result shown in Figure 2(b) is the case of optimal train dispatching without the consideration of station layout. It is clear that four trains require to stop and pass station 4 during time period 110 to 120. If the station layout is the case shown in Figure 1(a), it is impossible to implement this result. The result with consideration of station layout is the case shown in Figure 2(c). It is clear that there is no more dispatching conflict. However, there is long delay for some trains at some stations even in the optimal result.

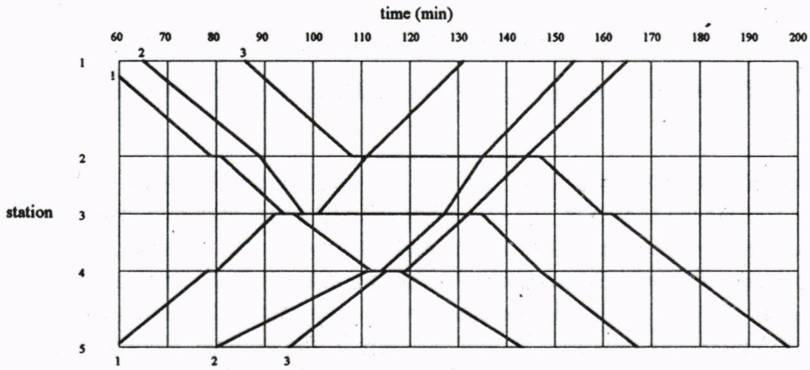


(a) The ideal schedule



O : capacity conflict of station layout

(b) The optimal dispatching without the consideration of station layout



(c) The optimal dispatching with the consideration of station layout

Figure 2 : The Train Digram of Five Stations and Six Trains

3.2 Model Application

After the model validity test, the model was used to 'study the operational characteristics of mixed train services. First, the relationship between mixed operation and average delay is considered. All experimentations were done in a study of 9 stations and six trains. For example, as the result shown in Figure 3 : the average delay for the operation of low speed train only , train speed = 80 km/hr, is 3.88 minutes per train-kilometer; and it is 3.22 minutes per train-kilometer for the operation of high speed train only, train speed = 120 km/hr. However, if there is one low speed train after each high speed train at the origin station, the optimal average delay is much higher than the mean of 3.88 and 3.22. That is, an increase in the number of speed classes might make a big trouble for the construction of tiemtable and train digram, and it might result in a decrease in the utilization of line capacity. Moreover, as the graph shown in Figure 3, the trouble of mixed operation for single track line is much severe than that for double track line or the rail line of double track with two-way signal.

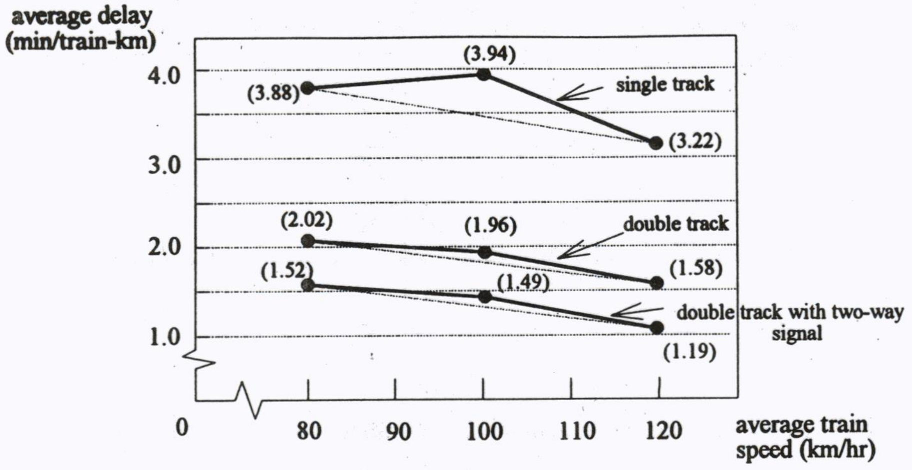


Figure 3 : Average Delay for Various Rail Line Layouts

(Distance between stations = 40km, and headway = 30 minutes.)

With the consideration of two speed classes, the relationship between average delay and speed difference is shown in Figure 4. For the first case, high and low speed trains have respectively the speeds of 120 km/hr and 80 km/hr. The optimal average delay for the mixed operation is 3.94 minutes per train-kilometer. Then, the speed difference is decreased so that the high and low speed trains respectively have the speeds of 120 km/hr and 90 km/hr. As the result shown in Figure 4, the optimal average delay of this case for the mixed operation is 3.79 minutes per train-kilometer. Therefore, a decrease in speed difference might result in a decrease of average delay for the mixed operation.

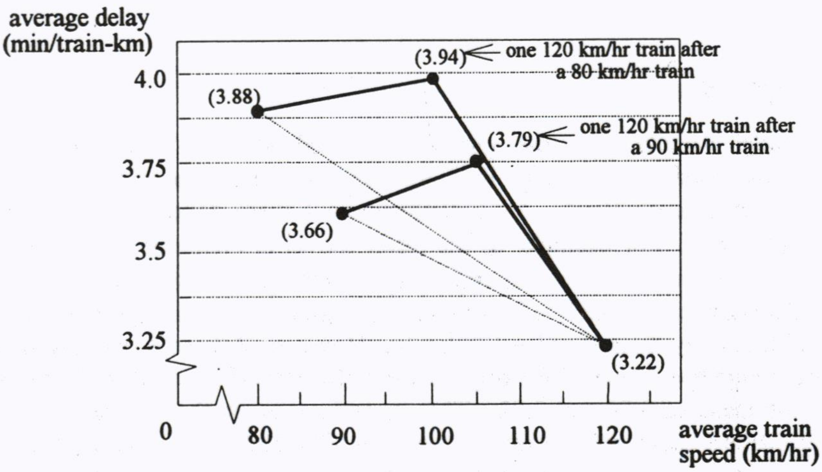


Figure 4 : Average Delay for Various Plans of Train-Mix

(Station distance = 40km, rail line = single track, and headway = 30 minutes.)

In this study, only station is considered as the meet point for trains. The relationship between average delay and the distance between stations for the mixed operation is shown in Figure 5. It is clear that an increase in station distance might result in an increased in optimal average delay. It is reasonable that the longer the station distance is, the longer the travel and waiting time is required for two conflict trains to solve its conflict. Furthermore, if the headway is considered as an explanatory variable for optimal average delay in the case of mixed operation, as the result shown in Figure 6, an decrease in headway might decrease the number of conflicts as well as the optimal average delay.

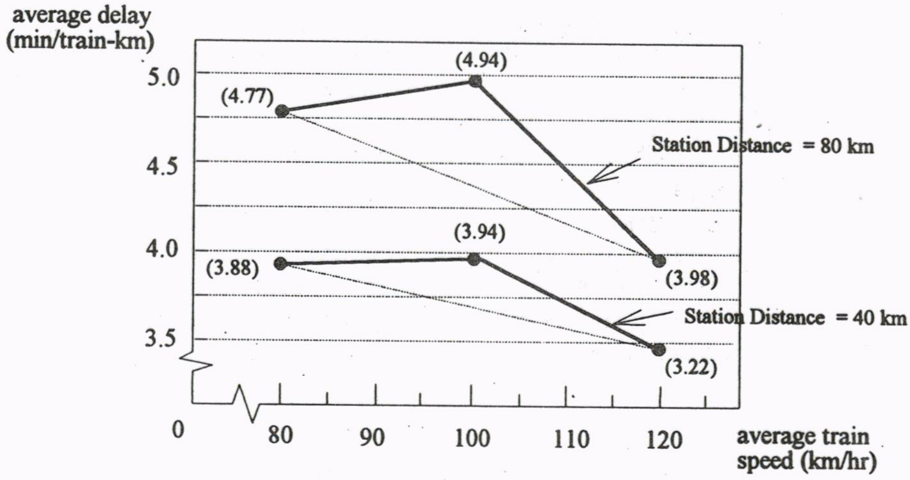


Figure 5 : Average Delay for Different Distances between Stations

(Rail line = single track, and headway = 30 minutes.)

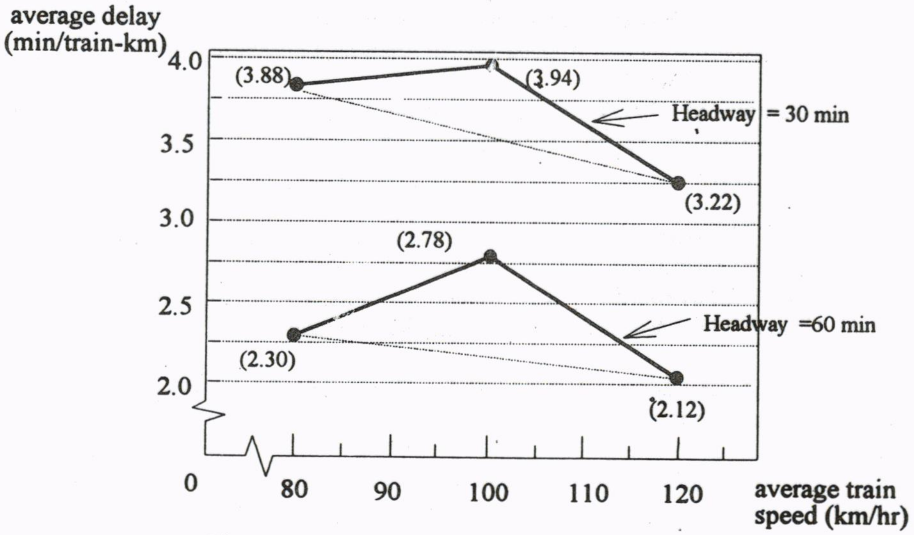


Figure 6 : Average Delay for Various Headways

(Station distance = 40km, and rail line = single track.)

5. CONCLUDING REMARKS

A mixed integer programming model is described in section 2 for a representation of the optimal train dispatching problem. It is a useful addition to the arsenal of techniques for practitioners who wish to tackle the train dispatching problem. However, it is necessary to verify the validity of the model carefully before any implementation, since it is a very complex formulation for a very complex problem. Besides solving the model for optimal timetable and train digram, the train dispatching model is a tool to investigate the operational characteristics of a train service plan. In section 3, testing results of the model are used to show the relationship between average delay and many factors of mixed operation.

Taiwan Railway Administration (TRA) has a big trouble in traffic planning and controling. As described in section 1, TRA uses a lot of mixed operation, such as the operation of freight and passenger trains and the operation of seven classes of train speeds. Moreover, some parts of TRA's rail line are single track sections, the speed difference in TRA's train classes is large, TRA's average station distance is in general short, TRA's operation density is high, and TRA's operation headway is small. As the testing results shown in section 3, all the factors listed above have an effect on the increase of average delay. Therefore, it is very difficult to operate TRA's rail line effectively. An examination on the demand and supply of train services is necessary to be done, in order to develop an appropriate strategy for long-run resource investment and short-run resource utilization.

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