Abstract: In this study, a risk metrization method intended for a railway transport system in the cold region is developed. The railway transport system is assumed to be developed by applying PFI system in this study. The risk metrization method applied to this study is based on VaR (Value at Risk). Three factors are used as the risk variables of the railway transport system, i.e., the population changes in the region where people can use the railway transport system, the price changes and the seasonal variations in the number of passengers. As the results, the following two conclusions are obtained. I) The expected profits maximization control by SPC makes not only expected profits but also the maximum amount of a loss larger than in the case where no optimization behaviors are introduced. II) The VaR minimization control makes not only the maximum amount of a loss but also the expected profits less.

Key Words: Risk Metrization, Value at Risk, Private Financial Initiative

1. INTRODUCTION

A legislation supporting PFI (Private Finance Initiative) has been established in 1999 and applied to public projects in Japan. The numbers of projects in the planning stage or under consideration is 187 projects, and in which SPC (Special Purpose Company) has been selected is 20 projects, at the time point of July in 2001. In applying PFI to the public project, it is important to metrize risk and return from the project for the purpose of assigning the risk between SPC and government, and determining grant money from the government.

In traditional studies with respect to metrizing risk, the risk is calculated by multiplying of probability of risk occurrence and maximum amounts of a loss. However, when the risk is actualized, whole risk should be coped with. Considering this situation, it is better not to discount the risk by multiplying the probability at this ratio risk happens. Risk analysis with respect to PFI includes economic evaluations of benefit and profits because the projects applied PFI are high public ones, on the other hand, SPC chases up profits from the projects. There are many studies dealing with economic benefit analysis under uncertainty (Johanson 1993, Ueda 1997, Tatano 1988). The studies dealing with the profits from PFI project are carried out from many view points, for example, Real Option Approach, demand prediction and so on (Fujita 1999, Otazawa 2001 and 2002, Takase 2002). However, there are a few studies that analyze the risk of PFI project based on real data. In other words, because most of these studies deal with risk or volatility of profits as parameter, it is difficult to metrize realistic risk of the projects.

In this study, a risk metrization method intended for a monorail transport system in the cold region is developed. The monorail transport system is assumed to be developed by applying
PFI system. The risk metrization method applied to this study is based on VaR (Value at Risk) method that is widely used in the financial facilities. This study targets only on the risks that occur in an operational term but on a construction term. Three factors are used as the risk factors of the monorail transport system, i.e., the population changes in the region where people can use the monorail transport system, the price changes and the seasonal variations in the number of passengers. In the case of cold region, because there are large number of passengers who change transport mode according to air temperature, the seasonal variations in the number of passengers should be considered as a risk factor. The population changes and the seasonal variations in the number of passengers have an influence on total number of passengers per a year, and the price changes have an influence on the operational costs per a year. In addition, it is assumed that SPC of the monorail transport system has to pay back construction costs including interest loaned from the financial facilities every year. However, PFI system with government’s debt guarantee is also assumed in this study because the project is high public one. This is expressed as grant money from government in the model. The profits of the SPC are determined by these relationships in this study.

Because three risk factors mentioned above have uncertainties, the profits of the project have also uncertainty. In this study, the uncertainty is expressed as a variance or a volatility, and the profits of SPC are expressed as a normal distribution with an average and a variance. The average and the variance of risk factors are calculated by using real data observed in the past. On the other hand, SPC can change the fare of the monorail transport system to maximize the expected profits or to minimize the risk of the project considering the number of passengers who change transport mode according to the fare changes. Two optimal behaviors of the SPC with respect to fare changes are introduced in this study. This study calculates the risk value of the monorail transport system planned in ISHIKARI City of Hokkaido, Japan as a test case.

2. MONORAIL DEVELOPMENT PLAN IN ISHIKARI CITY

ISHIKARI City is located at the north part of SAPPORO City and undertakes a role of a commuter town of SAPPORO City. The numbers of population in ISHIKARI City and SAPPORO City are 55,000 and 1,822,992, respectively. The main public transport system from ISHIKARI City to SAPPORO City is bus system. However, delay in arrival of the bus system, which happens not only in the rush but also in winter season when the deterioration in surface traffic environment happens, is a big problem in ISHIKARI City. Urban policies in ISHIKARI City are to build up coordination with SAPPORO City and to improve public transport system.

From the background of public transport system in ISHIKARI City mentioned above, a monorail transport system that enlinks between SAPPORO City and ISHIKARI City has been discussed by the institutions concerned. Fig.1 shows a plan of the monorail system discussed. The length of the monorail system is 15.2km and the monorail system consists of 13 stations. The monorail system is connected to a subway station where passengers can transfer to subway bound for the center of SAPPORO. However, because the monorail development projects carried out in the past by joint public-private venture had little prospect of breaking even, and the risk assignments between the government and the joint public-private venture were not cleared, the monorail development plan in ISHIKARI City has not been carried out at present.

3. VALUE at RISK

In this study, a risk metrization method intended for a monorail transport system in the cold
The risk metrization method applied to this study is based on VaR (Value at Risk) that is widely used in the financial facilities to measure the value of risk (Yamashita 2000).

The definition of VaR is that the maximum amount of a loss in the future that can happen at some probabilities. VaR means downside risk different from traditional risk, i.e. multiplication of probability of risk occurrence and maximum amounts of a loss. It is assumed that the rate of profitability is exposed to some risk factors and each risk factor is expressed as a normal distribution, as a result, the rate of profitability is also expressed as a normal distribution. Especially in the delta method, the rate of profitability is synthesized from a variance-covariance matrix of each risk factor. VaR is calculated as expected rate of profitability minus multiplication of a confidence coefficient ($\theta$) and standard deviation rate of profitability (Fig. 2). It is assumed that the probability of risk occurrence equals 1%, and then the confidence coefficient equals 2.33.

4. PROFITS MODEL OF MONORAILY TRANSPORT SYSTEM

4.1 Risk Factors

The elongation percentage of population in ISHIKARI City, the fluctuation in prices and the seasonal variations in the number of passengers are assumed to be risk factors with respect to the profits of monorail transport system in this study. The elongation percentage of population and the seasonal variations in the number of passengers affect total number of passengers for a year. The fluctuation in prices affects the annual costs of the monorail system.

It is natural to consider the number of passengers who use monorail system increases when the number of population in the area of surrounding the monorail system. Fig. 3 shows the annual elongation percentage of population in ISHIKARI City. Let us consider the relationship between the number of population and the number of passengers. Let $T_0$ (people/day) and $r_1^k$ (%) denote the number of daily passengers who use the monorail system independent of seasons, and the elongation percentage of population in ISHIKARI City during $k$ years, respectively. We assume that total number of passengers per a day after $k$ years from base year when the monorail transport system is opened up, i.e., $T_k$, is expressed by $T_0 (1+R_1^k)$, where $R_1^k = r_1^k/100$. 

![Figure 2. a conceptual diagram of VaR](image)

![Figure 3. annual elongation percentages of population in ISHIKARI City](image)

![Figure 4. paces of annual advance in prices in Japan](image)
Fig. 4 shows the pace of annual advance in prices in Japan. If there is a hike in prices, we assume that the costs of SPC for employment and operating increase in this study. Let $C_V \text{ (Yen/year)}$, $C_O \text{ (Yen/year)}$ and $r^k_2 \text{ (%/k years)}$ denote employment costs, operating costs in the base year when monorail transport system is opened up, and the rate of price changes during $k$ years, respectively. We assume that the employment costs and the operating costs in the $k$th year are expressed by $C_V(1+R^k_2)$ and $C_O(1+R^k_2)$, respectively. Where, $R^k_2$ is written as $r^k_2/100$.

![Figure 5. averaged highest temperatures of each month in SAPPORO](image)

$$p_m = -950 t_m + 252951 \quad (4.8^*) \quad (77^*) \quad R^2 = 0.76 \quad (1)$$

Fig.5 and Fig.6 show the averaged highest temperatures of each month in SAPPORO City, and the relationship between averaged number of daily passengers who use subway in SAPPORO City and the averaged highest temperatures of each month in SAPPORO City. These two figures indicate that the averaged highest temperatures are large enough to be considered as a risk factor of railway system in the cold region, because the variances of each month’s averaged highest temperatures are large and they make the seasonal variation in the number of passengers. Eq.(1) indicates a result of regression analysis between the averaged number of daily passengers in the $m$th month ($p_m$) and the averaged highest temperatures in the $m$th month ($t_m$), where, $m$s indicate 8 months except for the period of time from July to October when there is a little variance in the number of daily passengers. In the Eq.(1), the figures in a bracket indicate t-statics and the sign ** indicates that the parameters are significant at the probability of 99%. The number of samples used to estimate the parameters is 32, i.e., the number of samples for 4 years.

According to Eq.(1), if the averaged highest temperature decrease 1 degree centigrade, the number of daily passengers increases by 950 (people/day).

According to the demand analysis of the monorail system in ISHIKARI City (Fujita 1999), it is estimated that the number of passengers who does not change the transport mode according to seasons equals 46,200 (people/day) at the time of 1995. In this study, we assume that the number of daily passengers who use the railway system independent of seasons ($T^0$) equals 46,200 (people/day). On the other hand, the numbers of subway users who use Nanpoku Line in Sapporo independent of the seasons is 230,000 (people/day). It is assumed that a proportional relationship exists in these two figures, i.e., 46,200 and 230,000, it is determined that if the averaged highest temperature decrease 1 degree centigrade, the number of daily monorail users in ISHIKARI City increases at the rate of $\beta = 180 \text{ (people/day)}$.

Let $t_0^i$ and $t_{ki}$ denote the averaged highest temperature of $i$th month in the base year and the averaged highest temperature of $i$th month after $k$ years from base year, respectively, and $t_0^i$ is expressed by Eq.(2).

$$t_{ki} = t_0^i (1 + R_{3_{ki}}^i) \quad (2)$$

Where, $R_{3_{ki}}^i$ (%) indicates the percentage change in the averaged highest temperature of $i$th month during $k$ years.

### 4.2 Formulation of Profits From Monorail Transport System

On the assumptions mentioned above, the number of passenger in the $i$th month after $k$ years from base year ($T_{ki}$) is written as

$$T_{ki} = MT^0 (1 + R_{1_{ki}}^i) - M \beta_1 (1 + R_{1_{ki}}^i) (t - t_{ki})$$

$$= MT^0 (1 + R_{1_{ki}}^i) - M \beta_1 (1 + R_{1_{ki}}^i) [(t - t_0^i) - t_0^i R_{3_{ki}}^i], \quad (3)$$

where, $t$, $R_{1_{ki}}^i$, and $M$ indicate the averaged highest temperature from July to October (23.6 degrees centigrade), the rate of population change in the $i$th month after $k$ years from base year and the number of days in a month (=30), respectively. On the assumption that the population change happens only at the beginning of a year, $R_{1_{ki}}^i$ is written as

$$R_{1_{ki}}^i = R_1^k \quad \text{and} \quad \sum_{i=1}^{Y} R_{1_{ki}}^i = Y R_1^k , \quad (4)$$

where, $Y$ equals 12 months. On this condition, the number of passengers in the $k$th year is written as

$$T^k = \sum_{i=1}^{Y} T_{ki}$$

$$= YMT^0 (1 + R_1^k) - M (Yt \sum_{i=1}^{Y} t_0^i \beta_1) (1 + R_1^k) + M \beta_1 (1 + R_1^k) \left( \sum_{i=1}^{Y} t_0^i R_{3_{ki}}^i \right) \right). \quad (6)$$

If a hike in prices happens, it costs a lot of money for SPC. However, SPC can change the fare for the purpose of making profits. On the other hand, the advance in fare can make the passengers use another transport system. Fujita (1999) analyzed the relation between the increase in fare and the number of passengers who would use the monorail system in ISHIKARI City. According to this study, if SPC increase the fare by 1% from 250 yen, 0.068% of the passengers would change the transport mode. This analysis was carried out based on the assumption that the run interval of monorail system equals 15 minutes. In this study, the demands are not divided by age or purpose because of both the simplicity of formulation and the lack of the data available. However, when the demands are divided, the same framework described in this study can be applied. If the price elasticity is constant, the increase of $\Delta f^k$ yen from $f^0$ yen in the $k$th year will decrease the number of passengers by $\beta_2 (\Delta f^k / f^0)^{\beta_2}$, where, $\beta_2$ equals 0.068. As a result, Eq.(6) is rewritten as

$$T^k = \sum_{i=1}^{Y} T_{ki}$$

$$= YMT^0 \left( 1 + R_1^k - \beta_2 \frac{\Delta f^k}{f^0} \right) - M (Yt \sum_{i=1}^{Y} t_0^i) \beta_1 \left( 1 + R_1^k - \beta_2 \frac{\Delta f^k}{f^0} \right) + M \beta_1 \left( 1 + R_1^k - \beta_2 \frac{\Delta f^k}{f^0} \right) \left( \sum_{i=1}^{Y} t_0^i R_{3_{ki}}^i \right) \right).$$
The fare in the $k$th year is written as
\[ f^{k} = f^{0} + \Delta f^{k}. \] (8)

On the assumption that the fluctuation in prices happens only at the beginning of a year, the gross incomings ($P^k$) and the aggregate total costs ($C^k$) of SPC in the $k$th year are written as
\[ P^k = (f^{0} + \Delta f^{k})T^k \] and
\[ C^k = \frac{C_r r (1 + r)^N}{(1 + r)^N - 1} + (1 + R^k_2)C_V + (1 + R^k_2)C_O, \] respectively. Where, $C_r$ indicates the construction costs of the monorail system loaned from banking facilities and is paid back according to the principal and interest equal repayment during operational term ($N$ years) with interest rate ($r$ %). In this study, we assume that the employment costs and the operating costs are independent of weather conditions such as snow, because the monorail system considered here is suspended monorail that is difficult to be affected by weather conditions. In this case, the expected profits ($\mu^{k}_I$) in the $k$th year is written as
\[ \mu^{k}_I = E(P^k - C^k) \]
\[ = M(f^{0} + \Delta f^{k}) \left\{ YT^{0} - \beta_1(Yt - \sum_{i=1}^{Y} t_i) \right\} \mu^{k}_{R^1} - (C_V + C_O) \mu^{k}_{R^2} \]
\[ + M \beta_1(f^{0} + \Delta f^{k}) \left( 1 - \beta_2 \frac{\Delta f^{k}}{f^{0}} \sum_{i=1}^{Y} t_i \mu^{k}_{R^1} \right) \mu^{k}_{R^1} \mu^{k}_{R^1} + M \beta_1(f^{0} + \Delta f^{k}) \mu^{k}_{R^1} \right\} \mu^{k}_{R^1} \mu^{k}_{R^1} - \left\{ C_r r (1 + r)^N \right\} (1 + r)^N - 1 + C_V + C_O, \] (11)
where, the sign E and $\mu^{k}_I$ indicate the expectation operation and the expected value of risk variable $R$.  

5. RISK METRIZATION OF MONORAIL TRANSPORTATION SYSTEM

5.1 Risk Metrization

A variation of the expected incomings, i.e., $\Delta I^{k}$, when each risk variable changed in $\Delta R$ is calculated by applying one dimensional taylor expansion to Eq.(11) at the expected value vector of risk variables $u^{k} = (\mu^{k}_{R^1}, \mu^{k}_{R^1}, \mu^{k}_{R^1}, \cdots, \mu^{k}_{R^1})$, and is written as
\[ \Delta I^{k} = \frac{\partial I^{k}}{\partial R^1} \Delta R^1 + \frac{\partial I^{k}}{\partial R^2} \Delta R^2 + \frac{\partial I^{k}}{\partial R^3} \Delta R^3 + \epsilon^{k} \]
\[ = \left[ M(f^{0} + \Delta f^{k}) \left\{ YT^{0} - \beta_1(Yt - \sum_{i=1}^{Y} t_i (1 + \mu^{k}_{R^1})) \right\} \right] \Delta R^1 - (C_V + C_O) \Delta R^2 \]
\[ + \sum_{i=1}^{Y} \left\{ M \beta_1(f^{0} + \Delta f^{k}) \mu^{k}_{R^1} \right\} \mu^{k}_{R^1} \right\} \mu^{k}_{R^1} + \epsilon^{k}, \] (12)
where, $\epsilon^{k}$ indicates the error term. In the Eq.(12), $E^{k}_1$, $E^{k}_2$ and $E^{k}_3$ indicate the exposures of
risk variables, i.e., \( R_{1k} \), \( R_{2k} \) and \( R_{3ki} \), respectively.

\[
E_1^k = M(f^0 + \Delta f^k) \left[ YT^0 - \beta_1 \left( Yt - \sum_{i=1}^{r} t_i^i (1 + \mu_{R_1^k}^i) \right) \right]
\]

\[
E_2^k = -(C_v + C_o)
\]

\[
E_3^{ki} = M\beta_i(f^0 + \Delta f^k) \left[ \mu_{R_k^i}^i + 1 - \beta_2 \frac{\Delta f^k}{f^0} \right] y_i^i, \quad (i = 1, \ldots, Y)
\]

The expected profits \( \mu_t^k \) in the \( k \)th year is also written as

\[
\mu_t^k = u^k(E^k)^f + M(f^0 + \Delta f^k) \left[ 1 - \beta_2 \frac{\Delta f^k}{f^0} \right] \left( YT^0 - \beta_1 \left( Yt - \sum_{i=1}^{r} t_i^i \right) \right) - \left[ \frac{C_p r(1 + r)^Y}{(1 + r)^Y - 1} + C_v + C_o \right],
\]

where, the sign \( t \) and \( E^k \) indicates the transposition operation of matrix and the exposure vector, i.e., \( E^k = (E_1^k, E_2^k, E_3^{k1}, \ldots, E_3^{kY}) \). The present value of the expected profits during the operational term is expressed by Eq.(14).

\[
\mu_t = \sum_{k=0}^{N-1} \frac{\mu_t^k}{(1 + r)^k}
\]

In this study, because PFI system with government’s debt guarantee is assumed, a discount rate is the same as the interest rate. On the other hand, maximum amount of a loss from the monorail transport system in the \( k \)th year is written as

\[
\theta = \sqrt{E^kZ^k(E^k)^f} - \mu_t^k,
\]

\[
Z^k = \begin{pmatrix}
\sigma_{R_1^k}^2 & 0 & 0 \\
0 & \sigma_{R_2^k}^2 & 0 \\
0 & 0 & W^k
\end{pmatrix}
\]

\[
W^k = \begin{pmatrix}
\sigma_{R_1^k}^2 & \cdots & \text{Cov}(R_1^{k1}, R_1^{kY}) \\
\vdots & \ddots & \vdots \\
\text{Cov}(R_1^{kY}, R_1^{kY}) & \cdots & \sigma_{R_1^{kY}}^2
\end{pmatrix}
\]

where, \( \theta \) equals 2.33 if probability of a loss is assumed to be 1%, and if the expected profits of the project are expressed as a normal distribution. The present value of the maximum amount of a loss during operational term is expressed by Eq.(18).

\[
VaR_t^k = \sum_{k=0}^{N-1} \frac{VaR_t^k}{(1 + r)^k}
\]

5.2 Expected Profits Maximization Control

SPC can change the fare in the \( k \)th year corresponding to the variation of risk variables. As a first case, let us consider when SPC changes the fare to maximize the expected profits in each year. In this case, the objective function of SPC is written as

\[
\max_{\Delta f^k} \mu_t^k (\Delta f^k)
\]

\[
s.t. \Delta f^k > -f^0.
\]

Considering \( C^k \) is independent of \( \Delta f^k \), the objective function is written as

\[
E \{ P_k(\Delta f^k) \} = \left[ -\beta_2 f^0 (x^i)^2 + (1 + \mu_{R_1^k}^i + \beta_2) x^i \right] \left[ M \left( YT^0 - \beta_1 \left( Yt - \sum_{i=1}^{r} t_i^i C_{ki}^i \right) \right) \right]
\]

\[
= D^k f^0 \left[ -\beta_2 (x^i)^2 + (1 + \mu_{R_1^k}^i + \beta_2) x^i \right],
\]
where,
\[ x^k = 1 + \frac{\Delta f^k}{f^0}, \]
(22)
\[ C^{ki} = 1 + \mu_{ki}, \]
(23)
\[ D^k = M \left( YT^0 - \beta_{i} \left( Yt - \sum_{j=1}^{T} t_j C^{kj} \right) \right), \]
(24)
Considering \( D^k \) is positive number, Eq.(19) is a strongly concave function. As a result, optimal change of fare in the \( k \)th year \( \Delta f^k \) uniquely determined and is written as
\[ \Delta f^k = \frac{f^0(1 + \mu_{ki} - \beta_{i})}{2\beta_{i}}. \]
(25)

### 5.3 VaR Minimization Control

On the other hand, it may be natural that SPC changes the fare in the \( k \)th year so as to minimize the maximum amount of a loss, and this problem is written as
\[ \min_{\Delta f^k} VaR^k_\Delta (\Delta f^k) \]
(26)
s.t. \( \Delta f^k > f^0. \)
(27)
At first, let us consider the convexity of Eq.(26). Let the function inside of radical sign in the first tem of Eq.(15) be expressed by \( f(x^k) \). This function is written as
\[ f(x^k) = (f^0)^2 \sigma_{R^k}^2 (D^k x^k)^2 + F^k \]
\[ + (f^0)^2 \beta_{i} \left( \sum_{i=1}^{T} \sum_{j=1}^{T} \sigma_{R^k}^2 (B^i)^2 + \sum_{i=1}^{T} \sum_{j=1}^{T} 2 \text{Cov}(R^k_{i}, R^k_{j})B^i j \right)(x^k)^4 \]
\[ - 2(f^0)^2 \beta_{i} \left( \sum_{i=1}^{T} \sum_{j=1}^{T} \sigma_{R^k}^2 (B^i)^2 (C^i + \beta_{i}) + \sum_{i=1}^{T} \sum_{j=1}^{T} \text{Cov}(R^k_{i}, R^k_{j})B^i j (C^i + C^j + 2\beta_{i}) \right)(x^k)^3 \]
\[ + (f^0)^2 \left( \sum_{i=1}^{T} \sigma_{R^k}^2 (B^i)^2 (C^i + \beta_{i}) + \sum_{i=1}^{T} \sum_{j=1}^{T} 2 \text{Cov}(R^k_{i}, R^k_{j})B^i j (C^i + \beta_{i}) (C^j + \beta_{i}) \right)(x^k)^2, \]
where,
\[ B^i = M \beta_{i} t_0^i, \]
(29)
\[ B^i j = B^i B^j \text{ and } F^k = \sigma_{R^k}^2 (C^i + C^j)^2. \]
(30)
Again, Eq(28) is rewritten as
\[ f(x^k) = (f^0)^2 \beta_{i} \left( \sum_{i=1}^{T} \sum_{j=1}^{T} \sigma_{R^k}^2 (B^i)^2 + \sum_{i=1}^{T} \sum_{j=1}^{T} 2 \text{Cov}(R^k_{i}, R^k_{j})B^i j \right)(x^k)^4 \]
\[ - 2(f^0)^2 \beta_{i} \left( \sum_{i=1}^{T} \sum_{j=1}^{T} \sigma_{R^k}^2 (B^i)^2 (C^i + \beta_{i}) + \sum_{i=1}^{T} \sum_{j=1}^{T} \text{Cov}(R^k_{i}, R^k_{j})B^i j (C^i + C^j + 2\beta_{i}) \right)(x^k)^3 \]
\[ + (f^0)^2 \left( \sum_{i=1}^{T} \sigma_{R^k}^2 (B^i)^2 (C^i + \beta_{i}) + \sum_{i=1}^{T} \sum_{j=1}^{T} 2 \text{Cov}(R^k_{i}, R^k_{j})B^i j (C^i + \beta_{i}) (C^j + \beta_{i}) \right)(x^k)^2 + F^k \]
(32)
\[ = (f^0)^2 \beta_{i} \{ GW^k G^i \} (x^k)^4 \]
\[ + (f^0)^2 \{ HW^k (H^k)^t \} \} (x^k)^3 \]
\[ + (f^0)^2 \{ GW^k (H^k)^t \} \} (x^k)^2 \]
\[ + \alpha^k (x^k)^4 + \gamma^k (x^k)^3 + \eta^k (x^k)^2 + F^k, \]
where,
\[ G = (B_1^l, \cdots, B_Y^l), \quad H^k = (B_1^l (C^l_1 + \beta_1), \cdots, B_Y^l (C^l_Y + \beta_2)), \] (33)
\[ \alpha^k = (f^0 \beta_1)^2 (GW^k G')^t, \] (34)
\[ \gamma^k = -2 (f^0)^2 \beta_2 \{GW^k (H^k)\}' \] and (35)
\[ \eta^k = (f^0)^2 \{H^t W^k (H^k)\}' + \sigma^2_{\eta^k} (D^k)^2 \}. \] (36)

The condition that guarantees Eq.(26) is strongly convex function is \( f''(x^k) > 0 \), and this condition is written as
\[ \frac{8\alpha^k \eta^k - 3(\gamma^k)^2}{4\alpha^k} > 0. \] (37)

Because the value \( \alpha^k \) can be expressed as a quadratic form of covariance matrix with respect to \( R^k_3 \) and this covariance matrix is positive definite, \( \alpha^k \) is positive number. Next, let us consider the following Eq.(38).

\[ 8\alpha^k \eta^k - 3(\gamma^k)^2 = (f^0)^4 (\beta_2)^2 [8(GW^k G') [H^t W^k (H^k)'] + \sigma^2_{\eta^k} (D^k)^2] - 12 \{GW^k (H^k)\}'^2 ] \]
\[ = (f^0)^4 (\beta_2)^2 [8 \{GW^k (H^k)\}' [H^t W^k (H^k)'] - 12 \{GW^k (H^k)\}'^2 + 8\sigma^2_{\eta^k} (D^k)^2 (GW^k G')] \]
\[ = (f^0)^4 (\beta_2)^2 [8 \{GW^k (H^k)\}'^2 - 12 \{GW^k (H^k)\}'^2 + 8\sigma^2_{\eta^k} (D^k)^2 (GW^k G')] \]
\[ = 4(f^0)^4 (\beta_2)^2 [2\sigma^2_{\eta^k} (D^k)^2 - HW^k (H^k)'] \] (38)

The signs of Eq.(38) were numerically examined by using risk variables. As the results, the signs of Eq.(38) are all positive in every \( k \). This indicates that Eq.(28) is strongly convex function and the objective function expressed by Eq.(26) has unique solution. Each solution is obtained by solving \( \partial VaR^k_i (x^k) / \partial x^k = 0 \) with respect to \( x^k \), and this equation is written as
\[ \frac{\partial VaR^k_i (x^k)}{\partial x^k} = \frac{2(f^0 \beta_2)^2 GW^k G' (x^k)^3 - 3(f^0)^2 \beta_1 GW^k H' (x^k)^2}{[f^0 \beta_2)^2 GW^k G' (x^k)^3 - 2(f^0)^2 \beta_2 GW^k H' (x^k)^3 + \sigma^2_{\eta^k} (D^k)^2 (H^k)']^2 + \sigma^2_{\eta^k} (D^k)^2 (H^k)']^2} \]
\[ + D^k f^0 (2\beta_2 x^k - (1 + \mu^k_1 + \beta_2)) = 0. \] (39)

Indeed, an iterative method is required to determine \( x^k \).

### 6. WEIGHTING OF DATA OBSERVED

According to the idea of VaR, the prediction is carried out based on the data observed in the past. However, it is not appropriate way to deal with new and old one equally, because it is more difficult to predict future situation by using older data. We use weighted average and weighted variance of risk variables in this study.

The expected value \( (\mu^k_1) \) and the variance \( (\sigma^2_{\eta^k}) \) of risk variable \( (R^k_n) \) in the \( k \)th year are calculated by using the exponential weighted moving average method. It is assumed that the number of data with respect to risk variable \( R_n \) is \( M \), and that data is serialized in ascending order with respect to its age. The newest one is written as \( R_n^{(0)} \) and the oldest one is written as \( R_n^{(M-1)} \). A percentage change in risk variables during \( k \) years is written as
\[ R_n^{(j)} = \frac{R_n^{(M-j-1)} - R_n^{(M-j-k)}}{R_n^{(M-j-k)}}, \quad M-j-k \geq 0, \] (40)
where, \(j\) indicates the order with respect to the age of data, and \(j=0\) indicates that the data is the newest one. An exponential weighted moving average and an exponential weighted moving variance of percentage change in risk variables during \(k\) years are written as

\[
\mu_{R^j} = \sum_{j=0}^{M-k-1} w_j R_n^{k(j)} \quad \text{and} \quad \sigma^2_{R^j} = \frac{M - k + 1}{M - k} \sum_{j=0}^{M-k-1} \left( R_n^{k(j)} - \mu_{R^j} \right)^2,
\]

respectively. In the case of \(R_3\), an exponential weighted moving covariance of percentage change during \(k\) years is written as

\[
\text{Cov}(R_3^{w_j}, R_3^{w_l}) = \frac{M - k + 1}{M - k} \sum_{j=0}^{M-k-1} w_j (R_3^{w(j)} - \mu_{R^j})(R_3^{w(l)} - \mu_{R^l}), n \neq l,
\]

where, \(w_j\) indicates the weight with respect to \(j\)th data. Because \(w_j\) is in accordance with an exponential distribution, then \(w_j\) is written as

\[
w_j = \alpha^j (1 - \alpha), \quad 0 < \alpha < 1.0 \quad \text{and} \quad \sum_{j=1}^{J} w_j = 1.0,
\]

where, \(\alpha\) indicates an attenuation coefficient. The weight of newer data is larger than older one. The attenuation is calculated based on the idea of Morgan (Morgan (1995)) and is written as

\[
\alpha = \exp \left( \frac{\ln J}{\Omega_j} \right),
\]

where, \(J\) and \(\Omega_j\) indicate the number of data and an acceptable error value with respect to Eq.(45), respectively. If the acceptable error value is determined, \(\alpha\) is calculated by Eq.(46).

In this study, \(\alpha\) is determined by using 0.1% as an error value mentioned above.

7. RESULTS

In this section, the results of risk metrization of the monorail system in ISHIKARI City are showed. The following preconditions based on the existing study (Fujita 1999) are applied in the analysis.

<table>
<thead>
<tr>
<th>Table 1. preconditions in the analysis</th>
<th>item value</th>
</tr>
</thead>
<tbody>
<tr>
<td>operational term</td>
<td>25 years</td>
</tr>
<tr>
<td>initial fare</td>
<td>250 yen</td>
</tr>
<tr>
<td>construction costs</td>
<td>89 billion yen</td>
</tr>
<tr>
<td>employment costs</td>
<td>0.46 billion yen/year</td>
</tr>
<tr>
<td>operational costs</td>
<td>2.0 billion yen/year</td>
</tr>
<tr>
<td>discount rate / rate of interest</td>
<td>4.0%</td>
</tr>
</tbody>
</table>

Fig.7 shows the result of an analysis where the fare is fixed at 250 yen, i.e., no optimization behaviors of SPC and no money grants from government are introduced. In this case, the expected profits and the maximum amount of a loss during operational term are calculated as –39.6 billion yen and –108.4 billion yen, respectively.

Fig.8 and Fig.9 show the results of analysis where the expected profits maximization control and the VaR minimization control are introduced, respectively. According to the result applying the expected profits maximization control, the expected profits and the maximum amount of a loss during operational term are –12.7 billion yen and –136.0 billion yen, respectively. In this case, the expected profits are larger than in the case where no
optimization behaviors are introduced. However, the maximum amount of a loss during operational term is also larger than in the case where no optimization behaviors are introduced. On the other hand, in the result applying the VaR minimization control, the expected profits and the maximum amount of a loss during operational term are –49.0 billion yen and –106.0 billion yen, respectively. In this case, the expected profits are less than in the case where no optimization behaviors are introduced. However, the maximum amount of a loss during operational term is also less than in the case where no optimization behaviors are introduced. The fare changes shown in the Fig.8 and Fig.9 are calculated based on the condition that there are no regulations on fare changes. If the government controls the fare changes from the viewpoint of mass transit, the government has to pay grant money so as to equalize the expected profits with regulations and that without regulations because what SPC emphasizes most is expected profits. This grant money is also calculated by the results brought by changing the constraint expressed by Eq.(20) or Eq.(27). From these three figures, it is clarified that I) the expected profits maximization control makes not only the expected profits but also the maximum amount of a loss larger than in the case where no optimization behaviors of SPC are introduced, and II) the VaR minimization control makes not only the expected profits but also the maximum amount of a loss less.

![Figure 7](image1)

**Figure 7. a result of analysis where fare is fixed**

![Figure 8](image2)  
![Figure 9](image3)

**Figure 8. a result of analysis where the expected profits maximization control is introduced**  
**Figure 9. a result of analysis where the VaR minimization control is introduced**

In the results showed above, it is assumed that there is no grant money from the government. Because the government is a guarantee of debt, the government has to pay debt of the project back to the banking facilities in the last year of the operational term. Adversely, SPC would not enter into the project like this. In the following analyses, it is assumed that the government pays grant money in the first year so as to make the expected profits during operational term equal 0. However, risk averse SPC would not still enter into the project in this case, this analysis would provide information as possible for determining the appropriate amount of grant money from the government.

Fig.10 shows the result of an analysis, where the fare is fixed at 250 yen and the government pays grant money. Amount of grant money by which the expected profits during operational term become 0 is calculated as 38.0 billion yen. In this case, the maximum amount of a loss during operational term is calculated as –68.8 billion yen.

Fig.11 and Fig.12 show the results of analysis where the expected profits maximization control and the VaR minimization control, in which government pays grant money, are...
introduced, respectively. In the result of the expected profits maximization control, the amount of grant money is calculated as 12.2 billion yen. The maximum amount of a loss during operational term is calculated as –123.0 billion yen. On the other hand, in the result of the VaR minimization control, the amount of grant money is calculated as 47.0 billion yen. The maximum amount of a loss during the operational term by the project is calculated as –57.0 billion yen.

The expected profits maximization control is desirable for the private companies. However, because the project applied PFI is high public one, it is better for the SPC’s behavior to minimize VaR. The amount of grant money calculated under an assumption that SPC changes the fare so as to minimize VaR, i.e., 47.0 billion yen, happens to equal infrastructure costs of the project. Because the risk factors used in this study affect the cash flow of SPC, nobody can manage these risk factors effectively. However, SPC can respond these risk factors by changing the fare. On the other hand, if all of the risks used in this study were distributed to SPC, SPC would never enter into the project because of the possibility of debt default. The risk of debt default is distributed to the government, and this is expressed as grant money in this study. The rest of risks related to the cash flow are distributed to SPC. In the case where the grant money calculated in this study is paid to SPC in the first year, the amount of debt guarantee during operational term would be certainly paid back to the government by SPC. Actually, debt guarantee by the government is required because there are some years when the expected profits become negative number.

\[ \text{Figure 10. a result of analysis with grant money} \]

\[ \text{Figure 11. a result of the expected profits maximization control with grant money} \]

\[ \text{Figure 12. a result of the VaR minimization control with grant money} \]

8. CONCLUSIONS

In this study, a risk metrization method intended for a monorail transport system in the cold region is developed. The monorail transport system is assumed to be developed by applying PFI. In the case of cold region, because there are large number of passengers who change transport mode according to air temperature, the seasonal variations in the number of passengers are considered as a risk factor. The population changes and the price changes are also considered as risk factors in this study.

In the analysis on metrization of the risk of monorail system, two optimization behaviors by SPC of the project, i.e., the expected profits maximization control and the VaR minimization control are compared. The results are shown in Figures 11 and 12.

\[ \text{Journal of the Eastern Asia Society for Transportation Studies, Vol.5, October, 2003} \]
control, are introduced. As the results, it is clarified that I) the expected profits maximization control makes not only the expected profits but also VaR larger than in the case where no optimization behaviors are introduced, and II) the VaR minimization control makes not only VaR but also the expected profits less. The analyses, which are based on the assumption that the government pays grant money so as to make the expected profits during operational term equal 0, are also carried out in this study. It is concluded that the amount of grant money from government, i.e., 47.0 billion yen calculated based on the VaR minimization control and is equivalent to infrastructure costs of the project, is a reasonable share of the expenses by the government for the project.

In this study, the grant of money from government is determined so as to make the expected profits during operational term equal 0. However, risk averse SPC would not enter into the project in this case. In the future study, it has to be considered how much grant of money is required to motivate the risk averse SPC to enter into the project based on utility function.

REFERENCES


